Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. All students should therefore be advised to show their working

1. *[Maximum mark: 16]*

The function f is defined by $f: x \mapsto -0.5x^2 + 2x + 2.5$ $f(0): .5(0) + 2(0) + 2.5 = y$

(a) Write down

(i) $f'(x)$; $-x+2$

(ii) $f'(0)$. 2

[2 marks]

$0 = -x + 2$ $x = 2$ $x = 2$

(b) Let N be the normal to the curve at the point where the graph intercepts the y-axis. Show that the equation of N may be written as $y = -0.5x + 2.5$. *[3 marks]*

so $\perp$ $m = \frac{1}{2}$

$y = -.5x + 2.5$

Let $g: x \mapsto -0.5x + 2.5$.

(c) (i) Find the solutions of $f(x) = g(x)$.

(ii) Hence find the coordinates of the other point of intersection of the normal and the curve. *[6 marks]*

$(-5, -20)$

(d) Let R be the region enclosed between the curve and N.

(i) Write down an expression for the area of R.

$\int_{-5}^{0} (-.5x^2 + 2x + 2.5)dx - \int_{-5}^{0} -.5x + 2.5 \Rightarrow$

(ii) Hence write down the area of R. *[5 marks]*

Can use calc here

$.5x^2 + 2x + 2.5 = -.5x + 2.5$

$.5x^2 + 2.5x = 0$

$.5x(x+5) = 0$

$x = 0$ or $x = -5$

$f(-5) = .5(-5)^2 + 2(-5) + 2.5$

$-12.5 - 10 + 2.5 = -20$

$\int_{-5}^{0} -.5x^2 + 2.5x\,dx$

$\left. -\frac{1}{6}x^3 + 1.25x^2 \right]_{-5}^{0}$

$0 - \left(-\frac{1}{6}(-5)^3 + 1.25(-5)^2\right)$

$+20.83 + 31.25$

$\boxed{52.08}$

22xx-xxxx

2. *[Maximum mark: 18]*

A farmer owns a triangular field ABC. One side of the triangle, [AC], is 104 m, a second side, [AB], is 65 m and the angle between these two sides is 60°.

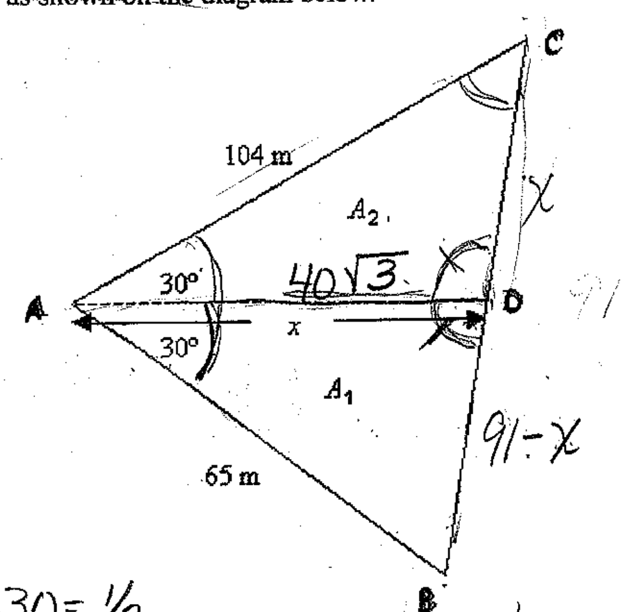
(a) Use the cosine rule to calculate the length of the third side of the field. *[3 marks]*

$CB^2 = 104^2 + 65^2 - 2(104)(65)\cos 60 = 91$

(b) Given that $\sin 60° = \frac{\sqrt{3}}{2}$, find the area of the field in the form $p\sqrt{3}$ where p is an integer. *[3 marks]*

$\frac{1}{2}(104)(65)\frac{\sqrt{3}}{2} = 1690\sqrt{3}$

Let D be a point on [BC] such that [AD] bisects the 60° angle. The farmer divides the field into two parts A_1 and A_2 by constructing a straight fence [AD] of length x metres, as shown on the diagram below.



$\sin 30 = \frac{1}{2}$

(c) (i) Show that the area of A_1 is given by $\frac{65x}{4}$.

$\frac{1}{2}ab\sin C = \frac{1}{2}(x)(65)(\frac{1}{2}) = \frac{65x}{4}$

(ii) Find a similar expression for the area of A_2. $\frac{104x}{4}$

(iii) **Hence**, find the value of x in the form $q\sqrt{3}$, where q is an integer. *[7 marks]*

(d) (i) Explain why $\sin A\hat{D}C = \sin A\hat{D}B$. Because they are supplementary

(ii) Use the result of part (i) and the sine rule to show that

$\frac{BD}{DC} = \frac{5}{8}$. *[5 marks]*

$\frac{\sin x}{65} = \frac{\sin(180-x)}{104}$ $\sin x = \frac{65}{104}$

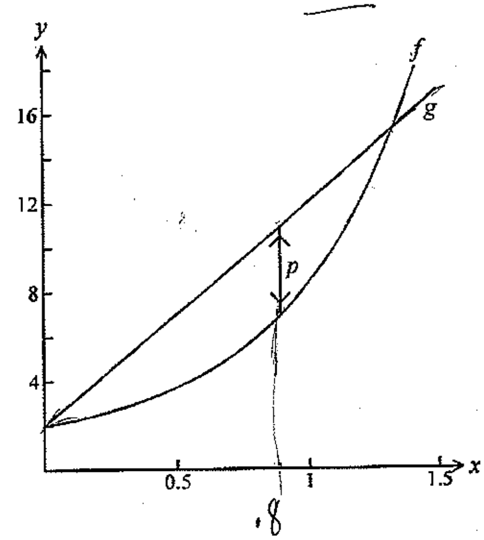
$\frac{104x}{4} + \frac{65x}{4} = 1690\sqrt{3}$ $169x = 6760\sqrt{3} =$

$x = 40\sqrt{3}$

3. *[Total mark: 22]*

Part A *[Maximum mark: 14]*

The diagram below shows the graphs of $f(x) = 1 + e^{2x}$, $g(x) = 10x + 2$, $0 \le x \le 1.5$



$5 = 1 + e^{2a}$

$4 = e^{2a}$

$\frac{\ln 4}{2} = \frac{2a}{2}$

$\frac{\ln 4}{2} = a$

(a) (i) Write down an expression for the vertical distance p between the graphs of f and g. $g(x) - f(x) = p$

(ii) Given that p has a maximum value for $0 \le x \le 1.5$, find the value of x at which this occurs. $10x + 2 - 1 - e^{2x} = 10x - e^{2x} + 1$ find deriv. *[6 marks]*

$\frac{\ln 5}{2}$

$10 - 2e^{2x} = 0$

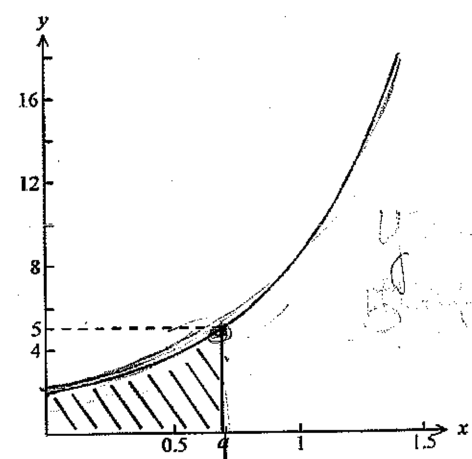
$-2e^{2x} = -10$

$\ln e^{2x} = \ln 5$

$2x = \ln 5$

$x = \frac{\ln 5}{2}$

The graph of $y = f(x)$ only is shown in the diagram below. When $x = a$, $y = 5$.



$2e^{2x}$

(b) (i) Find $f^{-1}(x)$.

(ii) **Hence** show that $a = \ln 2$. so $\frac{\ln 4}{2} = a$ $\quad \frac{1}{2}\ln 4 = \ln 4^{1/2} = \ln 2.$ *[5 marks]*

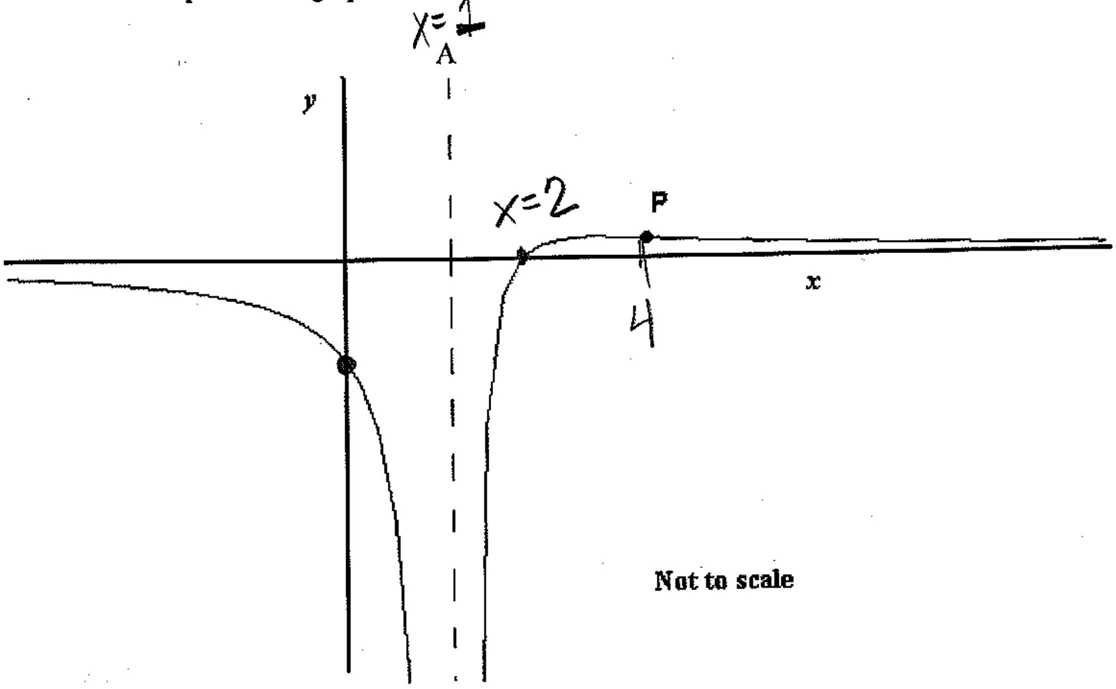
(c) The region shaded in the diagram is rotated through 360° about the x-axis. Write down an expression for the volume obtained. *[3 marks]*

$$\pi \int_0^{\ln 2} (1 + e^{2x})^2 \, dx$$

Part B *[Maximum mark: 8]*

Consider the function $h: x \mapsto \dfrac{x-2}{(x-1)^2}, x \neq 1$.

A sketch of part of the graph of h is given below.



The line (AB) is a vertical asymptote. The point P is a point of inflexion.

$-2x^2+6x-4$

(a) Write down the equation of the vertical asymptote. $x = 1$ *[1 mark]*

x^2-2x+1 $(x-2)(2x-2)$

(b) Find $h'(x)$, writing your answer in the form

$$\frac{a-x}{(x-1)^n}$$

$u'v - uv' \Rightarrow \dfrac{1(x-1)^2 - \boxed{(x-2)(2(x-1))}}{(x-1)^4} =$

$= \dfrac{-(x-3)}{(x-1)^3} = \dfrac{3-x}{(x-1)^3}$

$\dfrac{-x^2+4x-3}{(x-1)^4}$

where a and n are constants to be determined. *[4 marks]*

(c) Given that $h''(x) = \dfrac{2x-8}{(x-1)^4}$, calculate the coordinates of P. $(4, 2/9)$ *[3 marks]*

$\dfrac{-(x^2-4x+3)}{(x-1)^4} = \dfrac{-(x-3)\cancel{(x-1)}}{(x-1)^{\cancel{4}3}}$

$2x-8=0$

$2x=8$

$x=4$

$h(4) = \dfrac{4-2}{(4-1)^2} = \dfrac{2}{9}$

4. *[Maximum mark: 19]*

Bag A contains 2 red balls and 3 green balls. Two balls are chosen at random from the bag without replacement. Let X denote the number of red balls chosen. The following table shows the probability distribution for X.

X	0	1	2
$P(X = x)$	$\frac{3}{10}$	$\frac{6}{10}$	$\frac{1}{10}$

(a) Calculate $E(X)$, the mean number of red balls chosen. *[3 marks]*

$0 \cdot 3/10 + 1 \cdot 6/10 + 2 \cdot 1/10 = 6/10 + 2/10 = 8/10$

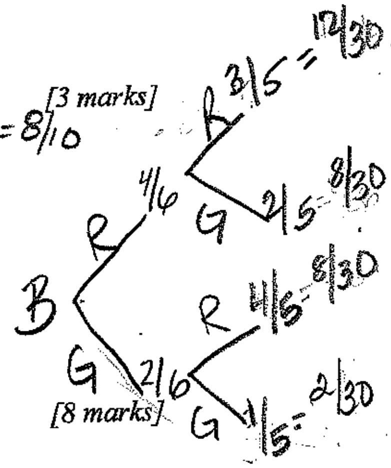
assumes No replacement

Bag B contains 4 red balls and 2 green balls. Two balls are chosen at random from bag B.

(b) (i) Draw a tree diagram to represent the above information, including the probability of each event.

(ii) Hence find the probability distribution for Y, where Y is the number of red balls chosen. *[8 marks]*

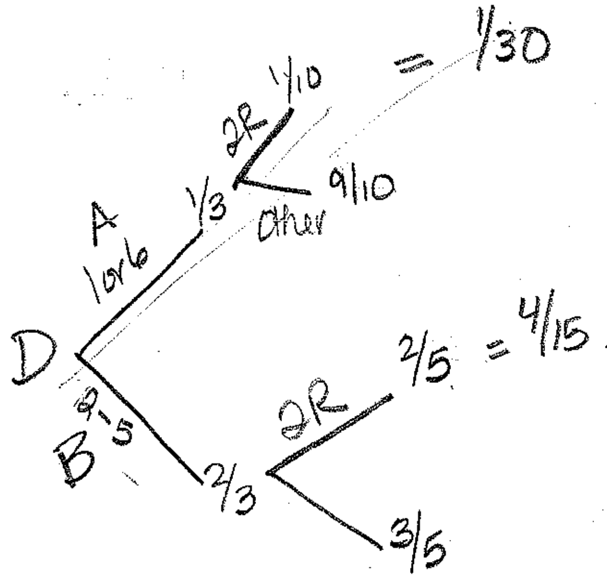
Y	0	1	2
$P(Y)$	$1/15$	$8/15$	$2/5$



A standard die with six faces is rolled. If a 1 or 6 is obtained, two balls are chosen from bag A, otherwise two balls are chosen from bag B.

(c) Calculate the probability that two red balls are chosen. $9/30$ *[5 marks]*

(d) Given that two red balls are obtained, find the conditional probability that a 1 or 6 was rolled on the die. *[3 marks]*



$$P(A|2R) = \frac{P(A \cap 2R)}{P(2R)} = \frac{1/30}{9/30} = 1/9$$

5. *[Maximum mark: 15]*

In this question, distance is in kilometers, time is in hours.
A balloon is moving at a constant height with a speed of $18\,\text{km h}^{-1}$, in the direction of the vector $\begin{pmatrix}3\\4\\0\end{pmatrix}$. $= \sqrt{9+16} = 5$ $5x = 18/5$ $x = 3.6$

At time $t=0$, the balloon is at point B with coordinates $(0, 0, 5)$.

(a) Show that the position vector $\boldsymbol{b}$ of the balloon at time t is given by

$$\boldsymbol{b} = \begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}0\\0\\5\end{pmatrix} + t\begin{pmatrix}10.8\\14.4\\0\end{pmatrix}.$$

$b = \begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}0\\0\\5\end{pmatrix} + t(3.6)\begin{pmatrix}3\\4\\0\end{pmatrix}$ *[6 marks]*

$b = \begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}0\\0\\5\end{pmatrix} + \begin{pmatrix}10.8\\14.4\\0\end{pmatrix}$

At time $t=0$, a helicopter goes to deliver a message to the balloon. The position vector $\boldsymbol{h}$ of the helicopter at time t is given by

$$\boldsymbol{h} = \begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}49\\32\\0\end{pmatrix} + t\begin{pmatrix}-48\\-24\\6\end{pmatrix}.$$

(b) (i) Write down the coordinates of the starting position of the helicopter. $(49, 32, 0)$

(ii) Find the speed of the helicopter. $\sqrt{(-48)^2+(-24)^2+6^2} = 54$ Kmh^{-1} *[4 marks]*

(c) The helicopter reaches the balloon at point R.

(i) Find the time ~~the helicopter takes to reach the balloon~~. $5/6$ hr.

(ii) Find the coordinates of R. $(9, 12, 5)$ *[5 marks]*

$49 - 48t = 0 + 10.8s$
$32 - 24t = 0 + 14.4s$
$0 + 6t = 5 + 0s$

$t = 5/6$

$49 - \cancel{48}^{8}(5/6) = 9$
$32 - \cancel{24}^{4}(5/6) = 12$
$0 + 6(5/6) = 5$