SECTION A

1. (a) $d = 3$ *A1* *N1*
[1 mark]

(b) (i) correct substitution into term formula *(A1)*
eg $u_{100} = 5 + 3(99)$, $5 + 3(100 - 1)$

$u_{100} = 302$ *A1* *N2*

(ii) correct substitution into sum formula *(A1)*
eg $S_{100} = \frac{100}{2}(2(5) + 99(3))$, $S_{100} = \frac{100}{2}(5 + 302)$

$S_{100} = 15350$ (accept 15 400 or 15 300) *A1* *N2*
[4 marks]

(c) correct substitution into term formula *(A1)*
eg $1502 = 5 + 3(n - 1)$, $1502 = 3n + 2$

$n = 500$ *A1* *N2*
[2 marks]

Total [7 marks]

2. (a) valid approach *(M1)*
eg $35 - 26$, $26 + p = 35$

$p = 9$ *A1* *N2*
[2 marks]

(b) (i) 26.7073170...
mean = 26.7 [26.7, 26.8] *A2* *N2*

(ii) recognizing that variance is $(\text{sd})^2$ *(M1)*
eg $11.021...^2$, $\sigma = \sqrt{\text{var}}$, $11.158...^2$

$\sigma^2 = 121.4753123...$
$\sigma^2 = 121$ [121,122] *A1* *N2*

Notes: If working shown and the candidate calculates mean without using frequencies (mean = 25), award full ***FT*** in (b)(ii) for $\sigma^2 = 200$ [199, 200].

If working shown and the candidate calculates mean using cumulative frequencies (mean = 33.7), award full ***FT*** in (b)(ii) for $\sigma^2 = 113$ [112, 113].

[4 marks]

3. (a) $p = 5,\ q = 7,\ r = 7$ (accept $r = 5$) *A1A1A1* *N3*

[3 marks]

(b) correct working *(A1)*

eg $\binom{12}{7} \times (3x)^5 \times (-2)^7$, 792, 243, -2^7, 24634368

coefficient of term in x^5 is -24634368 (accept -24600000 or -24700000) *A1* *N2*

Notes: Do not award the final *A1* for an answer that contains x.
If no working shown, award *N1* for $-24634368x^5$.

[2 marks]

Total [5 marks]

4. (a) (i) $A = \begin{pmatrix} -1 & -1 & 1 \\ 1 & 1 & 0 \\ -2 & -1 & 2 \end{pmatrix}$ *A1* *N1*

(ii) $A^{-1} = \begin{pmatrix} 2 & 1 & -1 \\ -2 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ *A2* *N2*

Note: Award *A1* for 6, 7 or 8 correct elements.

[3 marks]

(b) evidence of multiplying by A^{-1} (in any order) *(M1)*

eg $X = A^{-1}B$, BA^{-1}, one correct element

$X = \begin{pmatrix} 9 \\ -8 \\ 3.5 \end{pmatrix}$ (accept $x = 9,\ y = -8,\ z = 3.5$) *A2* *N3*

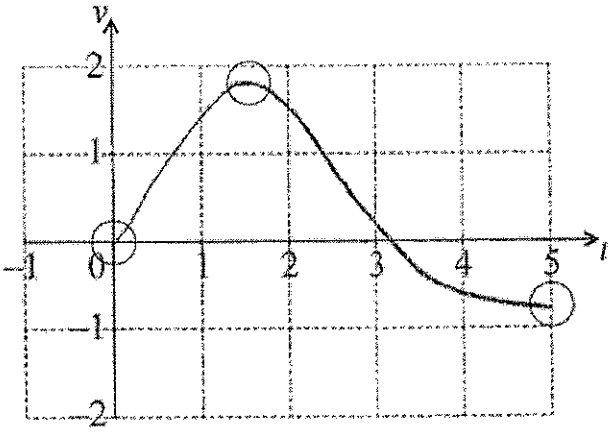
Notes: Award *A1* for two correct elements.
If no working shown, award *N2* for two correct elements.
If working shown, award *M1A1* if $X = \begin{pmatrix} 9 \\ -8 \\ 3.5 \end{pmatrix}$ follows from $X = BA^{-1}$.

[3 marks]

Total [6 marks]

5. **Note:** There was an unfortunate minor error on this question. Scripts looked at during standardisation did not indicate any major problems for candidates, but some may have been affected. Please look carefully at the work, and if you feel the error may have adversely affected the candidate, please keep a note of the scoris display and response ID, and send this information to the subject manager, einir.wyn.davies@ibo.org. Please include your examiner name and number. Thanks for your help and support with this. In the subject of the message, please put Q5 P2 TZ1.

(a)



A1A1A1 *N3*

Note: Award *A1* for approximately correct shape crossing x-axis with $3 < x < 3.5$ (check sketch carefully).
Only if this *A1* is awarded, award the following:
A1 for maximum in circle, *A1* for endpoints in circle.

[3 marks]

(b) (i) $t = 3.14159\ldots$
$t = \pi$(exact), 3.14 [3.14, 3.15] *A1* *N1*

(ii) recognizing distance is area under velocity curve *(M1)*
eg $s = \int v$, shading on diagram, attempt to integrate v

valid approach to find the total area *(M1)*
eg area A + area B, $\int v\,dt - \int v\,dt$, $\int_0^{3.14} v\,dt + \int_{3.14}^{5} v\,dt$, $\int |v|$

correct working with integration and limits (accept dx or missing dt) *(A1)*
eg $\int_0^{3.14} v\,dt + \int_5^{3.14} v\,dt$, $3.067\ldots + 0.878\ldots$, $\int_0^5 \left|e^{\sin t} - 1\right|$

3.94521...
distance = 3.95 [3.94, 3.95] *A1* *N3*

[5 marks]

Total [8 marks]

6. (a) (i) $k = 2$ *A1* *N1*

(ii) $p = -1$ *A1* *N1*

(iii) $q = 5$ *A1* *N1*

[3 marks]

(b) recognizing one transformation *(M1)*

eg horizontal stretch by $\frac{1}{3}$, reflection in x-axis,

A′ is $(2, -5)$ *A1A1* *N3*

Notes: If working shown, award ***M1A1A0*** for $(-2, 5)$.
If no working shown, award ***N1*** for $(-2, 5)$.

[3 marks]

Total [6 marks]

7. **Note:** All the marks in this question are implied, as candidates will choose to show different working, and will use differing approaches.
- Award the first 4 marks for finding one correct quartile (wherever seen).
- Seeing a second correct quartile achieves the next mark.
- The correct value for one quartile with the penultimate line and the correct final answer achieves full marks.

recognizing one quartile probability (may be seen in a sketch) *(M1)*

eg $P(X < Q_3) = 0.75$, 0.25

finding standardized value for either quartile *(A1)*

eg $z = 0.67448\ldots$, $z = -0.67448\ldots$

attempt to set up equation (must be with z – values) *(M1)*

eg $0.67 = \frac{Q_3 - 150}{10}$, $-0.67448 = \frac{x - 150}{10}$

one correct quartile *(A1)*

eg $Q_3 = 156.74\ldots$, $Q_1 = 143.25\ldots$

correct working *(A1)*

eg other correct quartile, $Q_3 - \mu = 6.744\ldots$

valid approach for IQR (seen anywhere) *(A1)*

eg $Q_3 - Q_1$, $2(Q_3 - \mu)$

13.4897949…

IQR = 13.5 [13.4, 13.5] (accept any notation that suggests the interval 143.25 - 156.74) *A1* *N4*

Note: If candidates use tables, their intermediate calculations may be slightly different depending on the level of accuracy used. Please check candidate working carefully

SECTION B

8. **Notes: Please read this carefully before marking this question.**

There was an unfortunate error on this question. Scripts looked at during standardisation did not indicate any major problems for candidates, but some may have been affected. Please look carefully at the work, and if you feel the error may have adversely affected the candidate, please keep a note of the scoris display and response ID, and send this information to the subject manager, einir.wyn.davies@ibo.org. Please include your examiner name and number. Thanks for your help and support with this. In the subject of the message, please put Q8 P2 TZ1.

This error has meant that there are many valid alternative approaches to this question, not addressed in this markscheme that could also lead to significant differences in answers. Examiners should check candidate work carefully and use discretion to award marks in line with the markscheme. However, most candidates seem to be using approach 1.

There may be slight differences in answers, depending on whether candidates use their 3 sf answers in subsequent parts. Do not penalize answers that are consistent with **their** working and check carefully for ***FT*** on acceptable answers.
Ignore incorrect or absence of units.

APPROACH 1 (mainly based on properties of triangles)

(a) evidence of choosing cosine rule *(M1)*
eg $c^2 = a^2 + b^2 - 2ab\cos C$, $CD^2 + AD^2 - 2 \times CD \times AD \cos D$

correct substitution *(A1)*
eg $10^2 + 7^2 - 2 \times 10 \times 7 \cos 125$, $149 - 140\cos 125$

15.1426781...
AC = 15.1 [15.1, 15.2] *A1* *N2*
[3 marks]

(b) (i) **METHOD 1**

evidence of choosing sine rule *(M1)*
eg $\frac{\sin A}{a} = \frac{\sin B}{b}$, $\frac{\sin A\hat{C}D}{AD} = \frac{\sin D}{AC}$

correct substitution *(A1)*
eg $\frac{\sin A\hat{C}D}{10} = \frac{\sin 125}{15.142...}$

$A\hat{C}D = 32.7487319...$
$A\hat{C}D = 32.7$ [32.7, 32.8] (32.9 [32.8, 32.9] from 15.1) *A1* *N2*

continued ...

Question 8 continued

METHOD 2

evidence of choosing cosine rule *(M1)*
eg $c^2 = a^2 + b^2 - 2ab\cos C$

correct substitution *(A1)*
eg $10^2 = 7^2 + 15.142...^2 - 2(7)(15.14...)\cos C$

$\hat{ACD} = 32.7487319...$
$\hat{ACD} = 32.7$ [32.7, 32.8] (33.1 [33.1, 33.2] from 15.1) *A1* *N2*

(ii) subtracting **their** $\hat{ACD}$ from 70 *(M1)*
eg $70 - \hat{ACD}$, $70 - 32.748...$

$\hat{ACB} = 37.2512680...$
$\hat{ACB} = 37.3$ [37.2, 37.3] (37.1 from 32.9), (36.9 from 33.1) *A1* *N2*
[5 marks]

(c) correct substitution *(A1)*
eg area $\Delta ADC = \frac{1}{2}(10)(7)\sin 125$

28.6703215...
area = 28.7 [28.6, 28.7] *A1* *N2*
[2 marks]

(d) **METHOD 1**

attempt to subtract *(M1)*
eg circle – ABCD, $\pi r^2 - \Delta ADC - \Delta ACB$

area $\Delta ACB = \frac{1}{2}(15.142...)(16)\sin 37.251$ (= 73.328...) (seen anywhere) *(A1)*

correct expression *(A1)*
eg $\pi(8)^2 - 28.670... - \frac{1}{2}(AC)(16)\sin \hat{ACB}$, $64\pi - 28.7 - 73.328$

99.0632979...
shaded area is 99.1 [99.0, 99.1], (99.5 [99.4, 99.5] from using rounded values) *A1* *N3*
[4 marks]

continued ...

Question 8 continued

METHOD 2

attempt to subtract *(M1)*

eg circle – ABCD, $\pi r^2 - \Delta ADC - \Delta ACB$

$\text{area}\,\Delta ACB = \frac{1}{2}(15.142\ldots)\left(\sqrt{16^2 - 15.142\ldots^2}\right) (= 39.122\ldots)$ (seen anywhere) *(A1)*

correct expression *(A1)*

eg $\pi(8)^2 - 28.670\ldots - \frac{1}{2}(15.142\ldots)(AB)$, $64\pi - 28.7 - 39.122$

133.2694663…

shaded area is 133 [133, 134] (132, [132,133] from using rounded values) *A1* *N3*

[4 marks]

Total [14 marks]

continued ...

Question 8 continued

APPROACH 2 (mainly based properties of cyclic quadrilaterals)

(a) evidence of finding $A\hat{B}C$ *(M1)*

eg $180-125,\ 55$, opp angles in a cyclic quad are supplementary

correct substitution *(A1)*

eg $\sin 55 = \frac{AC}{16}$

13.106432...

$AC = 13.1$ [13.1, 13.2] *A1* *N2*

[3 marks]

(b) (i) **METHOD 1**

evidence of choosing sine rule *(M1)*

eg $\frac{\sin A}{a} = \frac{\sin B}{b}$, $\frac{\sin A\hat{C}D}{AD} = \frac{\sin D}{AC}$

correct substitution *(A1)*

eg $\frac{\sin A\hat{C}D}{10} = \frac{\sin 125}{13.106...}$

$A\hat{C}D = 38.682187...$

$A\hat{C}D = 38.7$ [38.6, 38.7] (38.7 [38.7, 38.8] from 13.1) *A1* *N2*

METHOD 2

evidence of choosing cosine rule *(M1)*

eg $c^2 = a^2 + b^2 - 2ab\cos C$

correct substitution *(A1)*

eg $10^2 = 7^2 + 13.106...^2 - 2(7)(13.106...)\cos C$

$A\hat{C}D = 48.8350123...$

$A\hat{C}D = 48.8$ [48.8, 48.9] *A1* *N2*

(ii) **METHOD 1**

subtracting **their** $A\hat{C}D$ from 70 *(M1)*

eg $70 - A\hat{C}D$, $70 - 38.6821...$, $70 - 48.8350...$

$A\hat{C}B = 31.3179...$

$A\hat{C}B = 31.3$ [31.3, 31.4] *A1* *N2*

METHOD 2

subtracting **their** $A\hat{C}D$ from 70 *(M1)*

eg $70 - A\hat{C}D$, $70 - 38.6821...$, $70 - 48.8350...$

$A\hat{C}B = 21.1649...$

$A\hat{C}B = 21.2$ [21.1, 21.2] *A1* *N2*

Question 8 continued

(c) correct substitution *(A1)*

eg area $\Delta ADC = \frac{1}{2}(10)(7)\sin 125$

28.6703215...
area = 28.7 [28.6, 28.7] *A1* *N2*
[2 marks]

(d) **METHOD 1**

attempt to subtract *(M1)*
eg circle – ABCD, $\pi r^2 - \Delta ADC - \Delta ACB$

area $\Delta ACB = \frac{1}{2}(13.1064...)(16)\sin 31.3179... = (54.5001...)$ (seen anywhere) *(A1)*

correct expression *(A1)*
eg $\pi(8)^2 - 28.670... - \frac{1}{2}(AC)(16)\sin A\hat{C}B$, $64\pi - 28.7 - 54.5001$

117.8917407...
shaded area is 118 [117, 118] *A1* *N3*

METHOD 2

attempt to subtract *(M1)*
eg circle – ABCD, $\pi r^2 - \Delta ADC - \Delta ACB$

area $\Delta ACB = \frac{1}{2}(13.1064...)(16)\sin 21.1649... = (37.8568...)$ (seen anywhere) *(A1)*

correct expression *(A1)*
eg $64\pi - 28.7 - 37.8568$

134.5350527...
shaded area is 135 [134, 135] *A1* *N3*

METHOD 3

attempt to subtract *(M1)*
eg circle – ABCD, $\pi r^2 - \Delta ADC - \Delta ACB$

area $\Delta ACB = \frac{1}{2}(13.1064)\left(\sqrt{16^2 - 13.1064...^2}\right)(= 60.140483...)$ (seen anywhere) *(A1)*

correct expression *(A1)*
eg $\pi(8)^2 - 28.670... - \frac{1}{2}(13.1046...)(AB)$, $64\pi - 28.7 - 60.1404$

112.2514468...
shaded area is 112 [112, 113] *A1* *N3*
[4 marks]

9. (a) 1.9607843...

$f(0) = \frac{100}{51}$ (exact), 1.96, [1.96, 1.97] *A1* *N1*

[1 mark]

(b) setting up equation *(M1)*

eg $95 = \frac{100}{1+50e^{-0.2x}}$, sketch of graph with horizontal line at $y = 95$

34.28231...

$x = 34.3$, [34.2, 34.3] *A1* *N2*

[2 marks]

(c) upper bound of y is 100 *(A1)*

lower bound of y is 0 *(A1)*

range is $0 < y < 100$ *A1* *N3*

[3 marks]

(d) **METHOD 1**

setting function ready to apply the chain rule *(M1)*

eg $100(1+50e^{-0.2x})^{-1}$

evidence of correct differentiation (must be substituted into chain rule) *(A1)(A1)*

eg $u' = -100(1+50e^{-0.2x})^{-2}$, $v' = (50e^{-0.2x})(-0.2)$

correct chain rule derivative *A1*

eg $f'(x) = -100(1+50e^{-0.2x})^{-2}(50e^{-0.2x})(-0.2)$

correct working clearly leading to the required answer *A1*

eg $f'(x) = 1000e^{-0.2x}(1+50e^{-0.2x})^{-2}$

$f'(x) = \frac{1000e^{-0.2x}}{(1+50e^{-0.2x})^2}$ *AG* *N0*

[5 marks]

continued ...

Question 9 continued

METHOD 2

attempt to apply the quotient rule (accept reversed numerator terms) ***(M1)***

eg $\frac{vu'-uv'}{v^2}$, $\frac{uv'-vu'}{v^2}$

evidence of correct differentiation inside the quotient rule ***(A1)(A1)***

eg $f'(x)=\frac{\left(1+50e^{-0.2x}\right)(0)-100\left(50e^{-0.2x}\times -0.2\right)}{\left(1+50e^{-0.2x}\right)^2}$, $\frac{100(-10)e^{-0.2x}-0}{\left(1+50e^{-0.2x}\right)^2}$

Note: Award ***A1*** for 0 or implied 0, and ***A1*** for $\left(50e^{-0.2x}\right)\times -0.2$.

any correct expression for derivative (0 may not be explicitly seen) ***(A1)***

eg $\frac{-100\left(50e^{-0.2x}\times -0.2\right)}{\left(1+50e^{-0.2x}\right)^2}$

correct working clearly leading to the required answer ***A1***

eg $f'(x)=\frac{0-100(-10)e^{-0.2x}}{\left(1+50e^{-0.2x}\right)^2}$, $\frac{-100(-10)e^{-0.2x}}{\left(1+50e^{-0.2x}\right)^2}$

$f'(x)=\frac{1000e^{-0.2x}}{\left(1+50e^{-0.2x}\right)^2}$ ***AG*** ***N0***

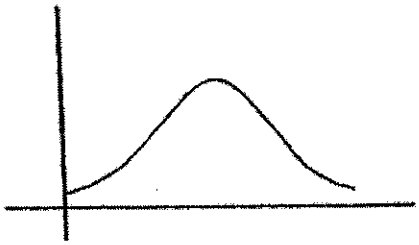
[5 marks]

Question 9 continued

(e) **METHOD 1**

sketch of $f'(x)$ *(A1)*

eg



recognizing maximum on $f'(x)$ *(M1)*

eg dot on max of sketch

finding maximum on graph of $f'(x)$ *A1*

eg $(19.6, 5)$, $x = 19.560\ldots$

maximum rate of increase is 5 *A1* *N2*

[4 marks]

METHOD 2

recognizing $f''(x) = 0$ *(M1)*

finding any correct expression for $f''(x)$ *(A1)*

eg $\dfrac{\left(1+50e^{-0.2x}\right)^2\left(-200e^{-0.2x}\right)-\left(1000e^{-0.2x}\right)\left(2\left(1+50e^{-0.2x}\right)\left(-10e^{-0.2x}\right)\right)}{\left(1+50e^{-0.2x}\right)^4}$

finding $x = 19.560\ldots$ *A1*

maximum rate of increase is 5 *A1* *N2*

[4 marks]

Total [15 marks]

10. (a) valid approach *(M1)*
eg $13 + \text{diameter}$, $13 + 122$

maximum height $= 135$ (m) *A1* *N2*
[2 marks]

(b) (i) $\text{period} = \frac{60}{2.4}$ *A1*

$\text{period} = 25$ (minutes) *AG* *N0*

(ii) $b = \frac{2\pi}{25}$ $(= 0.08\pi)$ (accept 14.4) *A1* *N1*
[2 marks]

(c) **METHOD 1**

valid approach *(M1)*
eg $\max - 74$, $|a| = \frac{135 - 13}{2}$, $74 - 13$

$|a| = 61$ (accept $a = 61$) *(A1)*

$a = -61$ *A1* *N2*
[3 marks]

METHOD 2

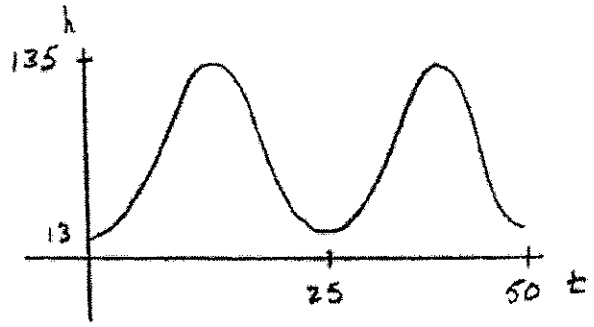
attempt to substitute valid point into equation for h *(M1)*
eg $135 = 74 + a\cos\left(\frac{2\pi \times 12.5}{25}\right)$

correct equation *(A1)*
eg $135 = 74 + a\cos(\pi)$, $13 = 74 + a$

$a = -61$ *A1* *N2*
[3 marks]

Question 10 continued

(d)



A1A1A1A1 *N4*

Note: Award *A1* for approximately correct domain, *A1* for approximately correct range, *A1* for approximately correct sinusoidal shape with 2 cycles.
Only if this last *A1* awarded, award *A1* for max/min in approximately correct positions.

[4 marks]

(e) setting up inequality (accept equation) *(M1)*
eg $h > 105$, $105 = 74 + a\cos bt$, sketch of graph with line $y = 105$

any **two** correct values for t (seen anywhere) *A1A1*
eg $t = 8.371\ldots$, $t = 16.628\ldots$, $t = 33.371\ldots$, $t = 41.628\ldots$,

Note: *FT* values of t from 61 are $t = 4.1289\ldots$, $t = 20.871\ldots$, $t = 29.1289\ldots$, $t = 45.8710\ldots$

valid approach *M1*

eg $\frac{16.628 - 8.371}{25}$, $\frac{t_1 - t_2}{25}$, $\frac{2 \times 8.257}{50}$, $\frac{2(12.5 - 8.371)}{25}$

0.330312304...
$p = 0.330$ [0.330, 0.331] *A1* *N2*

[5 marks]

Total [16 marks]