Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided.*

1. *[Maximum mark: 6]*

The following diagram shows triangle ABC.

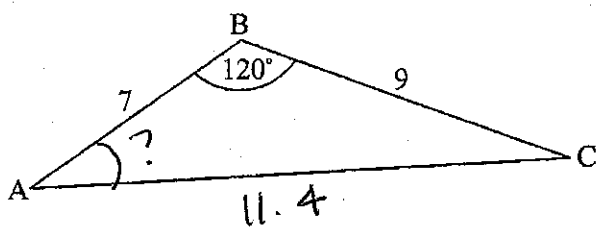


diagram not to scale

$AB = 7\,\text{cm}$, $BC = 9\,\text{cm}$ and $A\hat{B}C = 120°$.

(a) Find AC. *[3 marks]*

(b) Find $B\hat{A}C$. *[3 marks]*

a) $AC^2 = 7^2 + 9^2 - 2(7)(9)\cos 120$

$AC^2 = 49 + 81 - 126(-1/2)$

$= 130 + 63$

$AC^2 = 193$

$AC = 11.4$

b) $\frac{\sin 120}{11.4} = \frac{\sin B\hat{A}C}{9}$ $\approx 43.1°$

2. *[Maximum mark: 6]*

Let $f(x) = 3x^2$. The graph of f is translated 1 unit to the right and 2 units down. The graph of g is the image of the graph of f after this translation.

(a) Write down the coordinates of the vertex of the graph of g. *[2 marks]*

(b) Express g in the form $g(x) = 3(x-p)^2 + q$. *[2 marks]*

The graph of h is the reflection of the graph of g in the x-axis.

(c) Write down the coordinates of the vertex of the graph of h. *[2 marks]*

a) $(1, -2)$

b) $3(x-1)^2 - 2 = g(x)$

c) $(1, 2)$

3. *[Maximum mark: 5]*

In an arithmetic sequence $u_1 = 7$, $u_{20} = 64$ and $u_n = 3709$.

(a) Find the value of the common difference. *[3 marks]*

(b) Find the value of n. *[2 marks]*

a) $u_n = u_1 + (n-1)d$

$64 = 7 + (20-1)d$

$57 = 19d$

$3 = d$

$3709 = 7 + (n-1)3$

$3702 = 3n - 3$

$3705 = 3n$

$1235 = n$

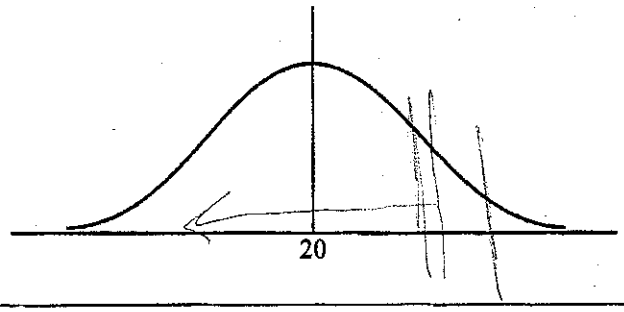
4. *[Maximum mark: 8]*

A random variable X is distributed normally with a mean of 20 and variance 9.

(a) Find $P(X \le 24.5)$. *[3 marks]*

(b) Let $P(X \le k) = 0.85$.

(i) Represent this information on the following diagram.



(ii) Find the value of k. $= 23.1$ *[5 marks]*

$z = \frac{24.5-20}{3} = 1.5$ 93.76%

$z = \frac{k-20}{3} \approx 23.1$

5. *[Maximum mark: 7]*

A box holds 240 eggs. The probability that an egg is brown is 0.05.

(a) Find the expected number of brown eggs in the box. *[2 marks]*

(b) Find the probability that there are 15 brown eggs in the box. *[2 marks]*

(c) Find the probability that there are at least 10 brown eggs in the box. *[3 marks]*

$240 \times .05$

$P(X=15)$

$C^{240}_{15}(.05)^{15}(.95)^{225}$

or use pdf in calc

$P(X \geq 10) = 1 - P(X \leq 10)$

use cdf in calc

6. *[Maximum mark: 6]*

Let $f(x) = \cos(x^2)$ and $g(x) = e^x$, for $-1.5 \le x \le 0.5$.

Find the area of the region enclosed by the graphs of f and g.

$$\int_{-1.5}^{1.5} \left(e^x - \cos(x^2)\right) = .574$$

use calc!

7. *[Maximum mark: 7]*

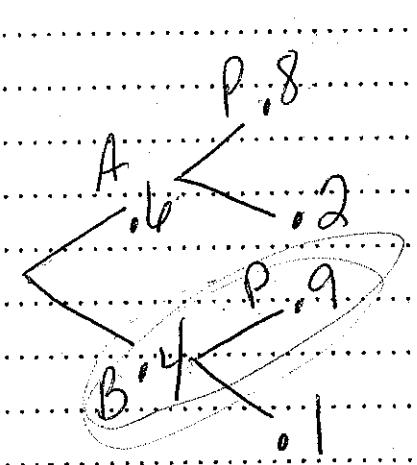
A company uses two machines, A and B, to make boxes. Machine A makes 60 % of the boxes.

80 % of the boxes made by machine A pass inspection.
90 % of the boxes made by machine B pass inspection.

A box is selected at random.

(a) Find the probability that it passes inspection. *[3 marks]*

(b) The company would like the probability that a box passes inspection to be 0.87. Find the percentage of boxes that should be made by machine B to achieve this. *[4 marks]*



$.6 \times .8 + .4 \times .9 = .84$

$P(P = .87)$

$.6 \times .8 +$

b) $.8(1-x) + .9(x) = .87$

$.8 - .8x + .9x = .87$

$.1x = .07 \quad = .7$

70%

*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

SECTION B

*Answer **all** questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 14]*

The following diagram shows a waterwheel with a bucket. The wheel rotates at a constant rate in an anticlockwise (counterclockwise) direction.

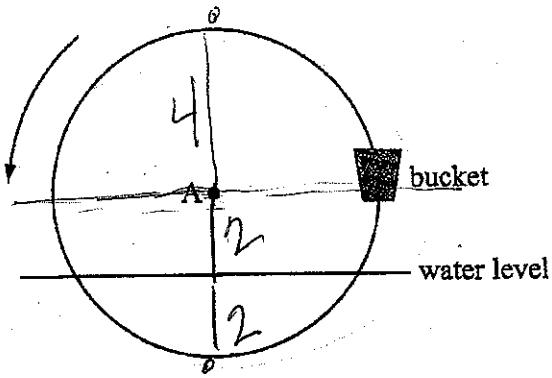


diagram not to scale

The diameter of the wheel is 8 metres. The centre of the wheel, A, is 2 metres above the water level. After t seconds, the height of the bucket above the water level is given by $h = a\sin bt + 2$.

(a) Show that $a = 4$. $6 - -2 = 8/2 = 4$ *[2 marks]*

The wheel turns at a rate of one rotation every 30 seconds.

(b) Show that $b = \frac{\pi}{15}$. $\frac{2\pi}{B} = 30$ $\frac{2\pi}{30} = B$ $\frac{\pi}{15} = B$ *[2 marks]*

In the first rotation, there are two values of t when the bucket is **descending** at a rate of $0.5\ \text{m s}^{-1}$.

(c) Find these values of t. find deriv. $= .5$ $4\sin(\pi/15)t + 2$ *[6 marks]*

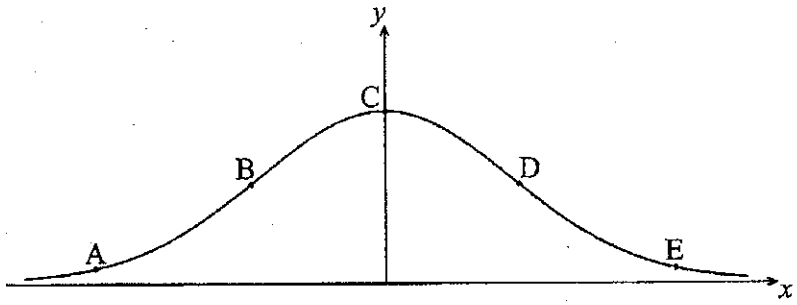
(d) Determine whether the bucket is underwater at the second value of t. *[4 marks]*

$f'(t) = -\frac{4\pi}{15}\cos(\pi/15)t = .5$ use calc to find values then plug 2nd into initial

*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

9. *[Maximum mark: 15]*

The following diagram shows the graph of $f(x) = e^{-x^2}$.



The points A, B, C, D and E lie on the graph of f. Two of these are points of inflexion.

(a) Identify the **two** points of inflexion. B, D *[2 marks]*

(b) (i) Find $f'(x)$. $-2xe^{-x^2}$

(ii) Show that $f''(x) = (4x^2 - 2)e^{-x^2}$. *[5 marks]*

(c) Find the x-coordinate of each point of inflexion. *[4 marks]*

(d) Use the second derivative to show that one of these points is a point of inflexion. *[4 marks]*

$(4x^2-2)e^{-x^2} = 0$

$4x^2 - 2 = 0$

$4x^2 = 2$

$x^2 = \frac{1}{2}$

$x = \pm\sqrt{1/2}$

then find values on either side

*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

10. *[Maximum mark: 16]*

Let $f(x) = \log_3 \frac{x}{2} + \log_3 16 - \log_3 4$, for $x > 0$.

(a) Show that $f(x) = \log_3 2x$. *[2 marks]*

(b) Find the value of $f(0.5)$ and of $f(4.5)$. *[3 marks]*

The function f can also be written in the form $f(x) = \frac{\ln ax}{\ln b}$.

(c) (i) Write down the value of a and of b.

(ii) Hence on graph paper, **sketch** the graph of f, for $-5 \le x \le 5$, $-5 \le y \le 5$, using a scale of 1 cm to 1 unit on each axis. use calc

(iii) Write down the equation of the asymptote. $y=0$, ~~x=~~ oops! *[6 marks]*

(d) Write down the value of $f^{-1}(0)$. *[1 mark]*

The point A lies on the graph of f. At A, $x = 4.5$.

(e) On your diagram, sketch the graph of f^{-1}, noting clearly the image of point A. *[4 marks]*

A' (2, 4.5)

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a) $\log_3\left(\frac{x}{2}\right) + \log_3\left(\frac{16}{4}\right) = \log_3\left(\frac{x}{2}\right) + \log_3(4) =$
$\log_3\left(\frac{x}{2}(4)\right) = \log_3 2x$

b) $\log_3(2(.5)) = \log_3 1 = 0$
$\log_3(2(4.5)) = \log_3 9 = 2$

c) $\ln 2x/3$ $a=2$ $b=3$

d) $y = \log_3 2x$ $\rightarrow$ $= 3^x = \cancel{3^{\log_3 2y}} = \frac{3^x}{2} = \frac{2y}{2} = y = \frac{3^x}{2}$
$x = \log_3 2y$

$f^{-1}(0) = \frac{3^0}{2} = 1/2$.