1. (a) $X \sim B(100, 0.02)$
$E(X) = 100 \times 0.02 = 2$ — A1 N1

(b) $P(X = 3) = \binom{100}{3}(0.02)^3 (0.98)^{97}$ — (M1)
$= 0.182$ — A1 N2

(c) **METHOD 1**
$P(X > 1) = 1 - P(X \le 1) = 1 - (P(X = 0) + P(X = 1))$ — M1
$= 1 - ((0.98)^{100} + 100(0.02)(0.98)^{99})$ — (M1)
$= 0.597$ — A1 N2

METHOD 2
$P(X > 1) = 1 - P(X \le 1)$ — (M1)
$= 1 - \ 0.40327$ — (A1)
$= 0.597$ — A1 N2

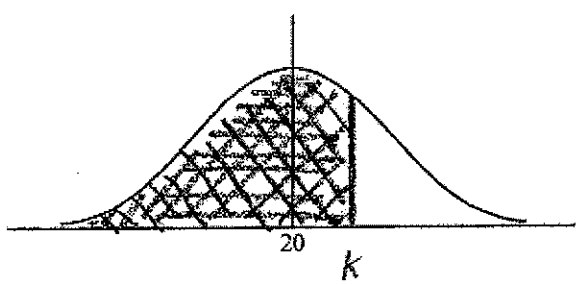
Note: *Award marks as follows for finding $P(X \ge 1)$, if working shown.*

$P(X \ge 1)$ — A0
$= 1 - P(X \le 2) = 1 - 0.67668$ — M1(FT)
$= 0.323$ — A1(FT) N0

[6]

2. (a) $\sigma = 3$ — (A1)
evidence of attempt to find $P(X \le 24.5)$ — (M1)
e.g. $z = 1.5$, $\frac{24.5 - 20}{3}$
$P(X \le 24.5) = 0.933$ — A1 N3 3

(b) (i)



A1A1 N2

Note: *Award A1 with shading that clearly extends to right of the mean, A1 for any correct label, either k, area or their value of k*

(ii) $z = 1.03(64338)$ (A1)
attempt to set up an equation (M1)
e.g. $\frac{k-20}{3} = 1.0364, \frac{k-20}{3} = 0.85$
$k = 23.1$ A1 N3 5

[8]

3. (a) (i) Attempt to find $P(3H) = \left(\frac{1}{3}\right)^3$ (M1)

$= \frac{1}{27}$ A1 N2

(ii) Attempt to find $P(2H, 1T)$ (M1)

$= 3\left(\frac{1}{3}\right)^2 \frac{2}{3}$ A1

$= \frac{2}{9}$ A1 N2

(b) (i) Evidence of using np $\left(\frac{1}{3} \times 12\right)$ (M1)

expected number of heads = 4 A1 N2

(ii) 4 heads, so 8 tails (A1)
E(winnings) = 4 × 10 – 8 × 6 (= 40 – 48) (M1)
= –$ 8 A1 N1

[10]

4. (i) $P(X > 3\,200) = P(Z > 0.4)$ (M1)

$= 1 - 0.6554 = 34.5\,\%$ $(= 0.345)$ (A1) (N2)

(ii) $P(2\,300 < X < 3\,300) = P(-1.4 < Z < 0.6)$ (M1)

$= 0.4192 + 0.2257$

$= 0.645$ (A1)

$P(\text{both}) = (0.645)^2 = 0.416$ (A1) (N2)

(iii) $0.7422 = \mathrm{P}(Z < 0.65)$ (A1)

$\frac{d - 3\,000}{500} = 0.65$ (A1)

$d = \$\,3\,325$ (= \$ 3 330 to 3 s.f.) (Accept \$3325.07) (A1) (N3)

[8]

5. (a) correct substitution into formula for E(X) (A1)

e.g. 0.05×240

E(X) =12 A1 N2 2

(b) evidence of recognizing binomial probability (may be seen in part (a)) (M1)

e.g. $\binom{240}{15}(0.05)^{15}(0.95)^{225}$, $X \sim \mathrm{B}(240, 0.05)$

$\mathrm{P}(X = 15) = 0.0733$ A1 N2 2

(c) $\mathrm{P}(X \leq 9) = 0.236$ (A1)

evidence of valid approach (M1)

e.g. using complement, summing probabilities

$\mathrm{P}(X \geq 10) = 0.764$ A1 N3 3

[7]

6. (a) Evidence of using the complement *e.g.* $1 - 0.06$ (M1)

$p = 0.94$ A1 N2

(b) For evidence of using symmetry (M1)

Distance from the mean is 7 (A1)

e.g. diagram, D = mean – 7

$D = 10$ A1 N2

(c) $\mathrm{P}(17 < H < 24) = 0.5 - 0.06$ (M1)

$= 0.44$ A1

E(trees) = 200×0.44 (M1)

$= 88$ A1 N2

[9]

7. (a) evidence of binomial distribution (may be seen in parts (b) or (c)) (M1)
e.g. np, 100×0.04

mean = 4 A1 N2

(b) $P(X = 6) = \binom{100}{6}(0.04)^6(0.96)^{94}$ (A1)

$= 0.105$ A1 N2

(c) for evidence of appropriate approach (M1)
e.g. complement, $1 - P(X = 0)$

$P(X = 0) = (0.96)^{100} = 0.01687...$ (A1)
$P(X \geq 1) = 0.983$ A1 N2

[7]

8. (a) 36 outcomes (seen anywhere, even in denominator) (A1)

valid approach of listing ways to get sum of 5, showing at least two pairs (M1)
e.g. (1, 4)(2, 3), (1, 4)(4, 1), (1, 4)(4, 1), (2, 3)(3, 2) , lattice diagram

$P(\text{prize}) = \frac{4}{36}\left(= \frac{1}{9}\right)$ A1 N3

(b) recognizing binomial probability (M1)
e.g. $B\left(8, \frac{1}{9}\right)$, binomial pdf, $\binom{8}{3}\left(\frac{1}{9}\right)^3\left(\frac{8}{9}\right)^5$

P(3 prizes) = 0.0426 A1 N2

[5]

9. (a) evidence of valid approach involving A and B (M1)
e.g. $P(A \cap \text{pass}) + P(B \cap \text{pass})$, tree diagram

correct expression (A1)
e.g. $P(\text{pass}) = 0.6 \times 0.8 + 0.4 \times 0.9$

$P(\text{pass}) = 0.84$ A1 N2 3

(b) evidence of recognizing complement (seen anywhere) (M1)

e.g. $P(B) = x$, $P(A) = 1 - x$, $1 - P(B)$, $100 - x$, $x + y = 1$

evidence of valid approach (M1)

e.g. $0.8(1 - x) + 0.9x$, $0.8x + 0.9y$

correct expression A1

e.g. $0.87 = 0.8(1 - x) + 0.9x$, $0.8 \times 0.3 + 0.9 \times 0.7 = 0.87$, $0.8x + 0.9y = 0.87$

70 % from B A1 N2 4

[7]

10. (a) symmetry of normal curve (M1)

e.g. $P(X < 25) = 0.5$

$P(X > 27) = 0.2$ A1 N2 2

(b) **METHOD 1**

finding standardized value (A1)

e.g. $\frac{27 - 25}{\sigma}$

evidence of complement (M1)

e.g. $1 - p$, $P(X < 27)$, 0.8

finding z-score (A1)

e.g. $z = 0.84\ldots$

attempt to set up equation involving the standardized value M1

e.g. $0.84 = \frac{27 - 25}{\sigma}, 0.84 = \frac{X - \mu}{\sigma}$

$\sigma = 2.38$ A1 N3 5

METHOD 2

set up using normal CDF function and probability (M1)

e.g. $P(25 < X < 27) = 0.3$, $P(X < 27) = 0.8$

correct equation A2

e.g. $P(25 < X < 27) = 0.3$, $P(X > 27) = 0.2$

attempt to solve the equation using GDC (M1)

e.g. solver, graph, trial and error (more than two trials must be shown)

$\sigma = 2.38$ A1 N3 5

[7]

11. (a) evidence of appropriate approach (M1)

e.g. 1 – 0.85, diagram showing values in a normal curve

$P(w \geq 82) = 0.15$ A1 N2

(b) (i) $z = -1.64$ A1 N1

(ii) evidence of appropriate approach (M1)

e.g. $-1.64 = \frac{x-\mu}{\sigma}, \frac{68-76.6}{\sigma}$

correct substitution A1

e.g. $-1.64 = \frac{68-76.6}{\sigma}$

$\sigma = 5.23$ A1 N1

(c) (i) $68.8 \leq \text{weight} \leq 84.4$ A1A1A1 N3

Note: *Award A1 for 68.8, A1 for 84.4, A1 for giving answer as an interval.*

(ii) evidence of appropriate approach (M1)

e.g. $P(-1.5 \leq z \leq 1.5)$, $P(68.76 < y < 84.44)$

P(qualify) = 0.866 A1 N2

(d) recognizing conditional probability (M1)

e.g. $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$

P(woman and qualify) = 0.25 × 0.7 (A1)

$P(\text{woman} \mid \text{qualify}) = \frac{0.25 \times 0.7}{0.866}$ A1

P(woman | qualify) = 0.202 A1 N3

[15]