Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.

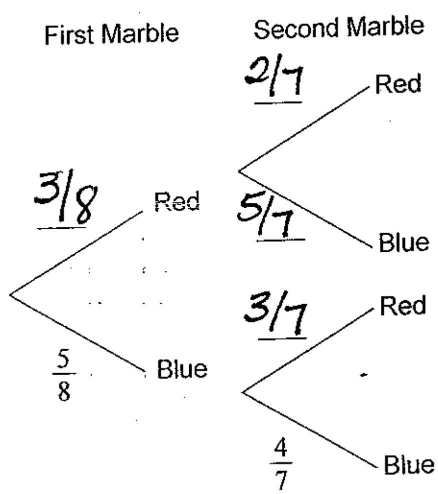
1. [Maximum mark: 6]

A bag contains eight marbles. Three marbles are red and five are blue. Two marbles are drawn from the bag without replacement.

(a) Write down the probability that the first marble drawn is red. [1]

$3/8$

(b) Complete the following tree diagram. [3]



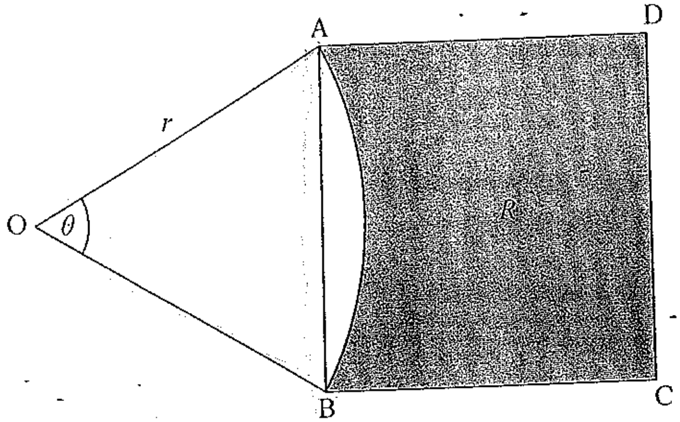
(c) Find the probability that both marbles are blue. [2]

$5/8 \cdot 4/7 = 20/56$

Do **not** write solutions on this page.

10. [Maximum mark: 16]

The following diagram shows a square ABCD, and a sector OAB of a circle centre O, radius r. Part of the square is shaded and labelled R.



$A\hat{O}B = \theta$, where $0.5 \le \theta < \pi$.

(a) Show that the area of the square ABCD is $2r^2(1-\cos\theta)$. [4]

(b) When $\theta = \alpha$, the area of the square ABCD is equal to the area of the sector OAB.

(i) Write down the area of the sector when $\theta = \alpha$.

(ii) Hence find α. [4]

(c) When $\theta = \beta$, the area of R is more than twice the area of the sector. Find all possible values of β. [8]

a) $(AB)^2 = r^2 + r^2 - 2r\cdot r\cdot \cos\theta$

$(AB)^2 = 2r^2 - 2r^2\cos\theta$

$= 2r^2(1-\cos\theta)$ ✓

b) $2\cancel{r^2}(1-\cos\alpha) = \frac{1}{2}\cancel{r^2}\alpha$

$2(1-\cos\alpha) = \frac{1}{2}\alpha$ (graph!)

$\alpha = .51$ rads.

c) $2\cancel{r^2}(1-\cos\beta) + \frac{1}{2}\cancel{r^2}\sin\beta - \frac{1}{2}\cancel{r^2}\beta > \cancel{r^2}\beta$

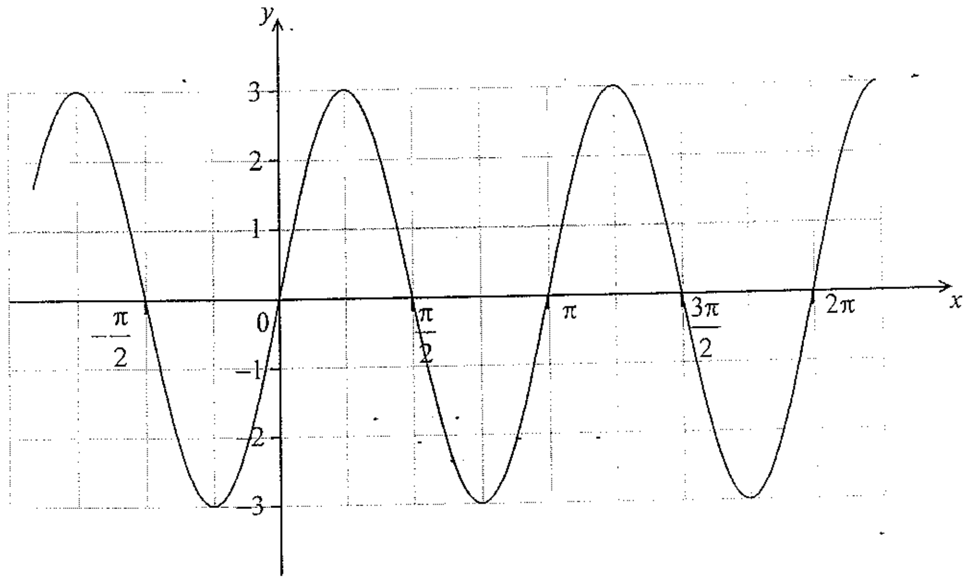
$2(1-\cos\beta) + \frac{1}{2}\sin\beta - \frac{1}{2}\beta > \beta$

graph!

$1.31 < \beta < 2.67$

2. [Maximum mark: 6]

Let $f(x) = a\sin bx$, where $b > 0$. The following diagram shows part of the graph of f.



(a) (i) Find the period of f.

(ii) Write down the amplitude of f. [3]

(b) (i) Write down the value of a.

(ii) Find the value of b. [3]

ai) π

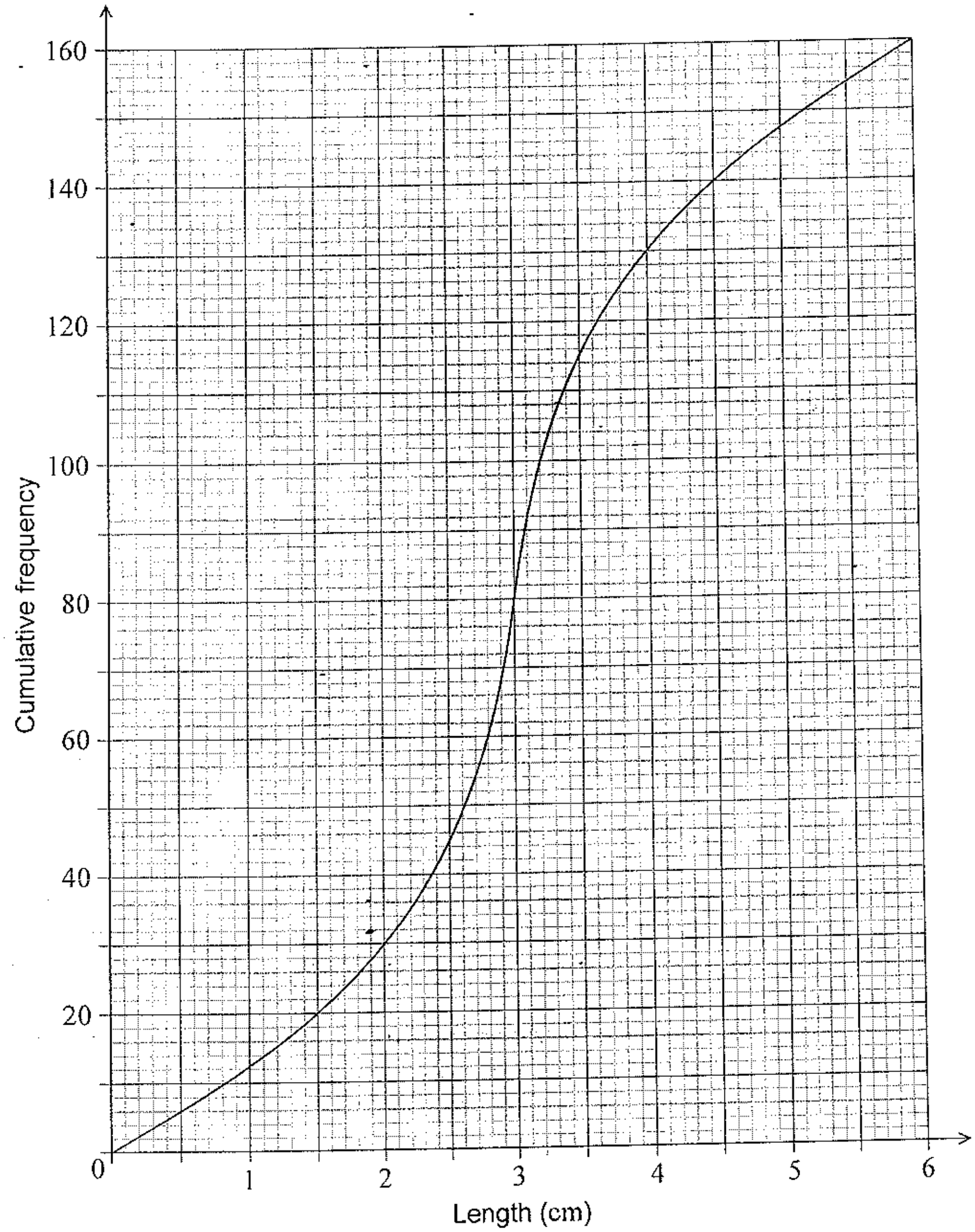
ii) 3

bi) 3

ii) 2

3. [Maximum mark: 6]

The following cumulative frequency diagram shows the lengths of 160 fish, in cm.



(This question continues on the following page)

(Question 3 continued)

(a) Find the median length. [2]

The following frequency table also gives the lengths of the 160 fish.

Length x cm	$0 \le x \le 2$	$2 < x \le 3$	$3 < x \le 4.5$	$4.5 < x \le 6$
Frequency	p	50	q	20

(b) (i) Write down the value of p.

(ii) Find the value of q. [4]

a) 3
b i) 30
ii) 60

4. [Maximum mark: 7]

Let $g(x) = \frac{\ln x}{x}$.

(a) Find $g'(x)$. [4]

(b) Find $\int g(x)\,\mathrm{d}x$. [3]

$\frac{\ln x}{x}$ $u = \ln x$ $v = x$

$u' = \frac{1}{x}$ $v' = 1$

$$\frac{x \cdot \frac{1}{x} - \ln x\,(1)}{x^2} = \frac{1 - \ln x}{x^2}$$

$$\int \frac{\ln x}{x} \Rightarrow \frac{1}{2}(\ln x)^2$$

5. [Maximum mark: 6]

Let $f(x) = e^{-2x}$.

(a) Write down $f'(x)$, $f''(x)$, and $f^{(3)}(x)$. [3]

(b) Find an expression for $f^{(n)}(x)$. [3]

e^{-2x}

$f'(x) = -2e^{-2x}$

$f''(x) = 4e^{-2x}$

$f'''(x) = -8e^{-2x}$

$f^{n}(x) = -2^{n}e^{-2x}$

6. [Maximum mark: 8]

Let $f(x) = ax^3 + bx$. At $x = 0$, the gradient of the curve of f is 3. Given that $f^{-1}(7) = 1$, find the value of a and of b.

$f'(x) = 3ax^2 + b$

$3 = 3a(0)^2 + b$

$3 = b$

$f(x) = ax^3 + 3$

$7 = a(1)^3 + 3$

$4 = a$

7. [Maximum mark: 7]

A bag contains black and white chips. Rose pays \$10 to play a game where she draws a chip from the bag. The following table gives the probability of choosing each colour chip.

Outcome	black	white
Probability	0.4	0.6

Rose gets no money if she draws a white chip, and gets $\$k$ if she draws a black chip. The game is fair. Find the value of k.

So $E(X)=10$

$\frac{.4k}{.4}=\frac{10}{.4}$

$k=25$

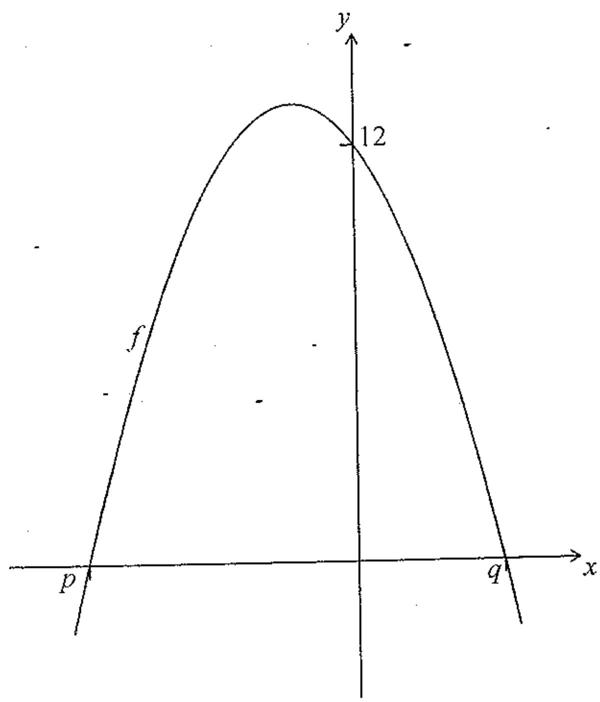
Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 15]

Let $f(x) = a(x+3)(x-1)$. The following diagram shows part of the graph of f.



The graph has x-intercepts at $(p, 0)$ and $(q, 0)$, and a y-intercept at $(0, 12)$.

(a) (i) Write down the value of p and of q. $p = -3$ $q = 1$

(ii) Find the value of a. -4 [6]

(b) Find the equation of the axis of symmetry of the graph of f. $x = -1$ [3]

(c) Find the largest value of f. $= 16$ [3]

The function f can also be written as $f(x) = a(x-h)^2 + k$.

(d) Find the value of h and of k. $f(x) = -4(x+1)^2 + 16$ [3]

Do **not** write solutions on this page.

9. [Maximum mark: 15]

Let P and Q have coordinates $(1, 0, 2)$ and $(-11, 8, m)$ respectively.

(a) Express $\overrightarrow{PQ}$ in terms of m. $\begin{pmatrix} -11-1 \\ 8-0 \\ m-2 \end{pmatrix} = \begin{pmatrix} -12 \\ 8 \\ m-2 \end{pmatrix}$ [2]

Let $\boldsymbol{a}$ and $\boldsymbol{b}$ be perpendicular vectors, where $\boldsymbol{a} = \begin{pmatrix} 1 \\ 1 \\ n \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} -3 \\ 2 \\ 1 \end{pmatrix}$ ✓

(b) Find n. $1 \cdot -3 + 1 \cdot 2 + n \cdot 1 = 0$ $\quad -3 + 2 + n = 0$ [4]

$-1 + n = 0$

$n = 1$

(c) Given that $\overrightarrow{PQ}$ is parallel to $\boldsymbol{b}$,

(i) express $\overrightarrow{PQ}$ in terms of $\boldsymbol{b}$; $4\vec{b}$

(ii) hence find m. [5]

$-12 = 4(-3)$

$8 = 4(2)$

$m-2 = 4(1)$

$m = 6$

In part (d), distance is in metres, time is in seconds.

(d) A particle moves along a straight line through Q so that its position is given by $\boldsymbol{r} = \boldsymbol{c} + t\boldsymbol{a}$.

(i) Write down a possible vector $\boldsymbol{c}$. $r = \begin{pmatrix} -11 \\ 8 \\ 6 \end{pmatrix} + t\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

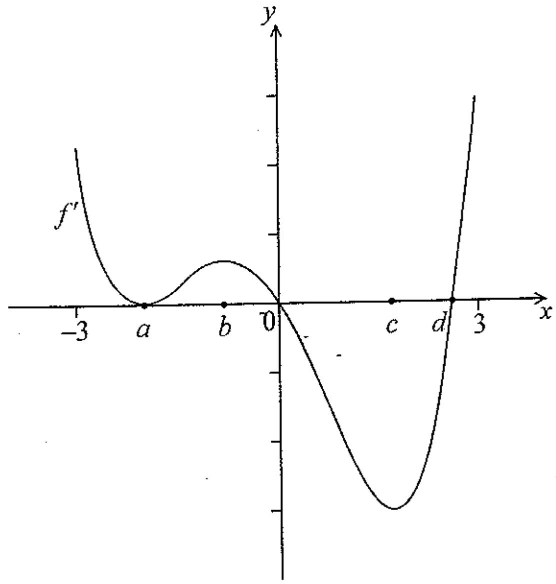
(ii) Find the speed of the particle. [4]

$\sqrt{1^2+1^2+1^2} = \sqrt{3}$ m/s^{-1}

Do **not** write solutions on this page.

10. [Maximum mark: 14]

Consider a function f with domain $-3 < x < 3$. The following diagram shows the graph of f', the **derivative** of f.



The graph of f' has x-intercepts at $x = a$, $x = 0$, and $x = d$. There is a local maximum at $x = b$ and local minima at $x = a$ and at $x = c$.

(a) Find all possible values of x where the graph of f is decreasing. [3]

(b) (i) Find the value of x where the graph of f has a local minimum.

(ii) Justify your answer. [3]

(c) The total area of the region enclosed by the graph of f' and the x-axis is 15. Given that $f(a) = 3$ and $f(d) = -1$, find the value of $f(0)$. [8]

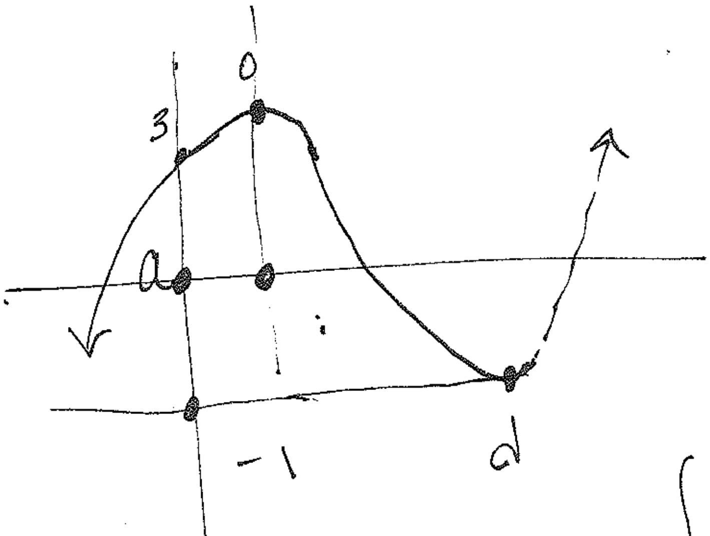
See attached

10) $0<x<d$

$x=0 \quad x=a \quad x=d$

$\int \quad = 15$

$f(a)=3 \quad f(d)=-1 \quad f(0)=?$



$\int f$

$$\int_a^0 f'(x)\,dx - \int_0^d f'(x)\,dx$$

$$f(x)\Big]_a^0 + f(x)\Big]_0^d = 15$$

$$\cancel{f(0)} - \overset{3}{f(a)} + \overset{-1}{f(d)} - \cancel{f(0)}$$

$$f(d) - f(a) = 15$$

$2+x=15$

$3+-1+x=15$

9) $\overrightarrow{PQ} = \begin{pmatrix} -12 \\ 8 \\ m-2 \end{pmatrix}.$

$1 \cdot -3 + 1 \cdot 2 + n \cdot 1 = 0$

$-3 + 2 + n = 0$

$n = 1$

$\cancel{(m-8\varepsilon)}$

$m - 2 = 4$

$m = 6$

4b

$m = 6$

$Q\,(-11, 8, 6)$

$r = \begin{pmatrix} -11 \\ 8 \\ 6 \end{pmatrix} + t \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad = | \quad | = \sqrt{3}$

$\int_a^0 f'(x) + \int_0^d |f'(x)|\, dx$

$f(a) = 3$

$f(d) = -1$

$f(0)$

$f(0) - 3 + |-1 - f(0)| = 15$

$f(0) - 3 + 1 + f(0) = 15$

$\frac{2f(0)}{2} = \frac{17}{2}$

$f(0) = 8.5$