1. The following diagram shows part of the graph of the function $f(x) = 2x^2$.

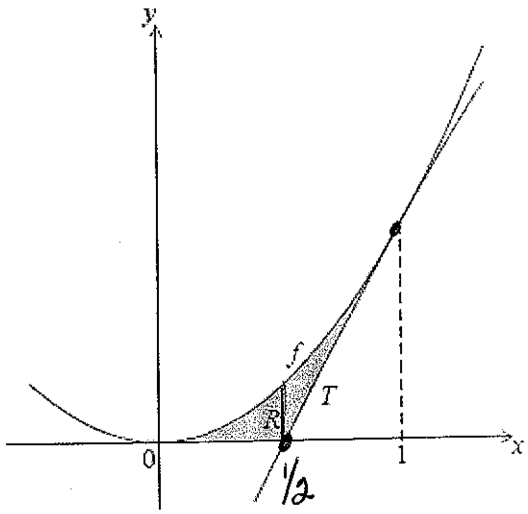


diagram not to scale

The line T is the tangent to the graph of f at $x = 1$.

(a) Show that the equation of T is $y = 4x - 2$.

(b) Find the x-intercept of T. $4x - 2 = 0 \quad 4x = 2 \quad x = 1/2$

(c) The shaded region R is enclosed by the graph of f, the line T, and the x-axis.

(i) Write down an expression for the area of R. $\int_0^{1/2} 2x^2 + \int_{1/2}^{1} 2x^2 - (4x-2)\,dx$

(ii) Find the area of R. $.083 \quad .0833 \quad = .1666$

2. (a) Find $\int \frac{1}{2x+3}\,dx$. $\frac{1}{2}\ln(2x+3) + C$

(b) Given that $\int_0^3 \frac{1}{2x+3}\,dx = \ln\sqrt{P}$, find the value of P. $\frac{1}{2}\ln(2x+3)\Big]_0^3$

$\frac{1}{2}\ln(9) - \frac{1}{2}\ln(3) = \frac{1}{2}\ln 3 = \ln 3^{1/2} = \ln\sqrt{3}$

3. Let $f'(x) = 12x^2 - 2$.

Given that $f(-1) = 1$, find $f(x)$. $\int 12x^2 - 2 \Rightarrow 4x^3 - 2x + C$

$1 = 4(-1)^3 - 2(-1) + C$

$1 = -4 + 2 + C \quad 1 = -2 + C \quad C = 3$

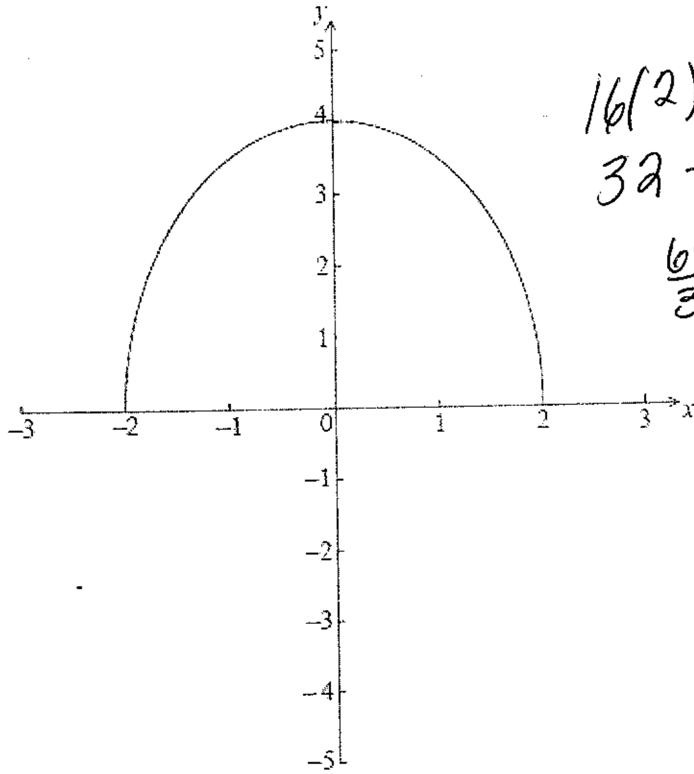
$f(x) = 4x^3 - 2x + 3$

4. (a) Find $\int_1^2 (3x^2 - 2)\,dx$.

$x^3 - 2x\Big]_1^2 = (2^3 - 4) - (1 - 2) \quad 4 + 1 = 5$

(b) Find $\int_0^1 2e^{2x}\,dx$. $\Rightarrow e^{2x}\Big]_0^1 \quad e^2 - e^0 = e^2 - 1$

5. The graph of $f(x) = \sqrt{16 - 4x^2}$, for $-2 \le x \le 2$, is shown below.



$16(2) - \frac{4}{3}(2)^3 - \left(16(-2) - \frac{4}{3}(-2)^3\right]$

$32 - \frac{32}{3}$ $\quad$ $(-32 + \frac{32}{3}$

$\frac{64}{3} \quad - \quad -\frac{96}{3} + \frac{32}{3}$

$-\frac{64}{3} =$

$\pi\left(\frac{128}{3}\right) = 42\frac{2}{3}\pi$

The region enclosed by the curve of f and the x-axis is rotated 360° about the x-axis.
Find the volume of the solid formed.

$\pi \int_{-2}^{2} (\sqrt{16-4x^2})^2 dx = \pi \int_{-2}^{2} 16 - 4x^2 \Rightarrow \pi\left[16x - \frac{4}{3}x^3\right]_{-2}^{2} =$

7. The graph of $y = \sqrt{x}$ between $x = 0$ and $x = a$ is rotated 360° about the x-axis.
The volume of the solid formed is 32π. Find the value of a.

$\pi \int_0^a (\sqrt{x})^2 dx = 32\pi \qquad \int_0^a x\, dx = 32 \qquad \frac{1}{2}x^2\Big]_0^a = 32$

$\frac{1}{2}a^2 = 32$

$a^2 = 64$

$a = 8$

8. *In this question s represents displacement in metres and t represents time in seconds.*

The velocity v m s^{-1} of a moving body is given by $v = 40 - at$ where a is a non-zero constant.

$\int 40 - at = 40t - \frac{1}{2}at^2$

(a) (i) If $s = 100$ when $t = 0$, find an expression for s in terms of a and t.

$s(t) = -\frac{1}{2}at^2 + 40t + 100$

(ii) If $s = 0$ when $t = 0$, write down an expression for s in terms of a and t.

$s(t) = -\frac{1}{2}at^2 + 40t$

Trains approaching a station start to slow down when they pass a point P. As a train slows down, its velocity is given by $v = 40 - at$, where $t = 0$ at P. The station is 500 m from P.

(b) A train M slows down so that it comes to a stop at the station.

(i) Find the time it takes train M to come to a stop, giving your answer in terms of a.

$40 - at = 0$

$40 = at$

$\frac{40}{a} = t$

(ii) Hence show that $a = \frac{8}{5}$.

$500 = -\frac{1}{2}a\left(\frac{40}{a}\right)^2 + 40\left(\frac{40}{a}\right)$

$500 = -\frac{1600}{2a} + \frac{1600}{a}$

(c) For a different train N, the value of a is 4.
Show that this train will stop **before** it reaches the station.

$500a = -800 + 1600$

$\frac{500a}{500} = \frac{800}{500} \qquad a = \frac{8}{5}$ ✓

$40 - 4t = 0$

$40 = 4t$

$10 = t$

$-\frac{1}{2}(4)(10)^2 + 40(10)$

$-200 + 400 = 200$

$200 < 500$

9. It is given that $\int_1^3 f(x)\,dx = 5$.

(a) Write down $\int_1^3 2f(x)\,dx$. $= 10$

(b) Find the value of $\int_1^3 (3x^2 + f(x))\,dx$.

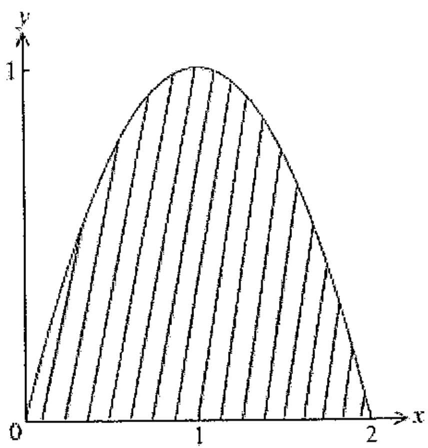
$\int_1^3 3x^2\,dx + \int_1^3 f(x)\,dx$

$x^3\Big]_1^3 + 5$

$3^3 - 1^3 + 5$

$27 - 1 + 5 = \boxed{31}$

10. A part of the graph of $y = 2x - x^2$ is given in the diagram below.



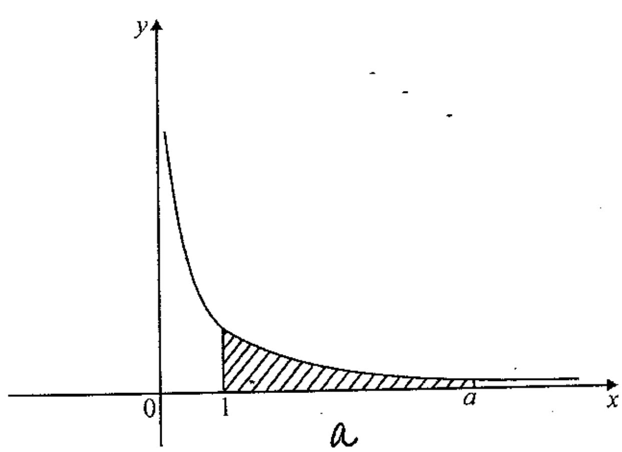
The shaded region is revolved through 360° about the x-axis.

(a) Write down an expression for this volume of revolution. $\pi\int_0^2 (2x-x^2)^2\,dx$ $1\frac{1}{3}\pi$

(b) Calculate this volume. $1\frac{1}{3}\pi$

Calculator

11. The diagram shows part of the graph of $y = \frac{1}{x}$. The area of the shaded region is 2 units.



$\int_1^a \frac{1}{x}\,dx = 2$

$\ln a = 2$

$a = e^2$

Find the exact value of a.

$\ln x\Big]_1^a = 2$

$\ln a - \ln 1 = \ln a - 0 = 2$

12. Given that $f(x) = (2x+5)^3$ find

(a) $f'(x)$;

(b) $\int f(x)\mathrm{d}x$.

13. A curve with equation $y = f(x)$ passes through the point (1, 1). Its gradient function is $f'(x) = -2x + 3$.

Find the equation of the curve.

this is #12

Working:

$\cancel{6}(2x+5)^2 =$ $\quad \frac{1}{\cancel{8}}(2x+5)^4$

Answer:

$6(2x+5)^2 = f'(x)$

(Total 4 marks)

$\int f(x)\,dx = \frac{1}{8}(2x+5)^4$

$(1,1) \quad f'(x) = -2x+3$

$\int -2x+3 \, dx = -x^2+3x+C$

$1 = -(1)^2 + 3(1) + C$

$1 = -1+3+C$

$1 = 2+C$

$-1 = C$

$f(x) = -x^2+3x-1$