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MATHEMATICS
STANDARD LEVEL
PAPER 1

Candidate session number

Tuesday 13 May 2014 (afternoon)

1 hour 30 minutes

Examination code

2	2	1	4	–	7	3	0	3

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics SL*** *formula booklet* is required for this paper.
- The maximum mark for this examination paper is *[90 marks]*.

11 pages

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 5]*

Let $f(x) = a(x-h)^2 + k$. The vertex of the graph of f is at $(2, 3)$ and the graph passes through $(1, 7)$.

(a) Write down the value of h and of k. *[2]*

(b) Find the value of a. *[3]*

1a) $2 = h \quad 3 = k$

b) $y = a(x-2)^2 + 3$

$7 = a(1-2)^2 + 3$

$4 = a + 3$

$1 = a$

2. *[Maximum mark: 7]*

In an arithmetic sequence, the third term is 10 and the fifth term is 16.

(a) Find the common difference. *[2]*

(b) Find the first term. *[2]*

(c) Find the sum of the first 20 terms of the sequence. *[3]*

$u_n = u_1 + (n-1)d$

$10 = u_1 + (3-1)d$

$16 = u_1 + (5-1)d$

$16 = u_1 + 4(3)$

$4 = u_1$

$16 = u_1 + 4d$

$-(10 = u_1 + 2d)$

$6 = 2d$

$3 = d$

$S_{20} = 20/2\,(2(4) + (19)3)$

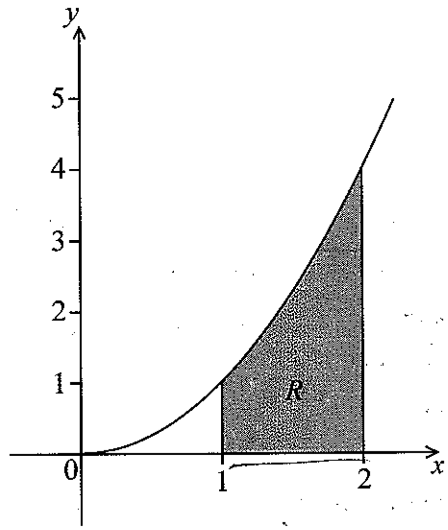
$= 10(8+57) = \boxed{650}$

3. *[Maximum mark: 6]*

Let $f(x) = x^2$.

(a) Find $\int_1^2 (f(x))^2 \mathrm{d}x$. *[4]*

(b) The following diagram shows part of the graph of f.



The shaded region R is enclosed by the graph of f, the x-axis and the lines $x = 1$ and $x = 2$.

Find the volume of the solid formed when R is revolved $360°$ about the x-axis. *[2]*

$\int_1^2 (x^4) = \left.\frac{1}{5}x^5\right]_1^2$

$\frac{1}{5} \cdot 32 - \frac{1}{5}(1)$

$= \frac{32}{5} - \frac{1}{5} = \frac{31}{5}$

b) $= \pi \frac{31}{5}$ or $\frac{31\pi}{5}$

4. *[Maximum mark: 6]*

(a) Write down the value of

(i) $\log_3 27$;

(ii) $\log_8 \frac{1}{8}$;

(iii) $\log_{16} 4$. *[3]*

(b) Hence, solve $\log_3 27 + \log_8 \frac{1}{8} - \log_{16} 4 = \log_4 x$. *[3]*

a i) = 3   ii) = −1   iii) = 1/2

b) $3 - 1 - 1/2 = 2.5$

$4^{2.5} = X$ $X = 32$

5. *[Maximum mark: 6]*

Celeste wishes to hire a taxicab from a company which has a large number of taxicabs. The taxicabs are randomly assigned by the company.

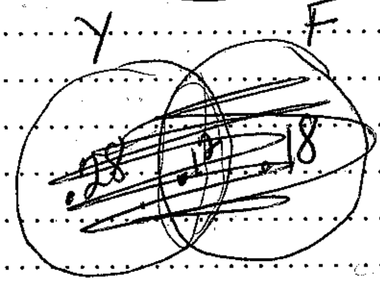
The probability that a taxicab is yellow is 0.4.
The probability that a taxicab is a Fiat is 0.3.
The probability that a taxicab is yellow or a Fiat is 0.6.

Find the probability that the taxicab hired by Celeste is **not** a yellow Fiat.

$P(Y \cup F) = P(Y) + P(F) - P(Y \cap F)$

$.6 = .4 + .3 - .12$

$P(Y') =$



$1 - .12 = .88$

$P(Y \cup F) =$

$P(Y \cap F) = P(Y) + P(F) - P(Y \cup F)$

$.3 + .4 - .6$

$= .1$ so

6. *[Maximum mark: 7]*

Let $\int_{\pi}^{a} \cos 2x \, dx = \frac{1}{2}$, where $\pi < a < 2\pi$. Find the value of a.

$\int_{\pi}^{a} \cos 2x \, dx = 1/2$ $\quad \int \cos 2x \, dx = \frac{1}{2} \sin 2x$

$\frac{1}{2} \sin 2x \Big]_{\pi}^{a} = 1/2$

$\frac{1}{2} \sin 2a - \left(\frac{1}{2} \sin 2\pi\right) = 1/2$

$\cancel{2}\left(\frac{1}{2} \sin 2a\right) - 0 = (1/2)^2$

$\sin^{-1}(\sin 2a) = \sin^{-1}(1)$

$2a = 90$

$a = 45$ or $\pi/4$

$\frac{\pi}{4} + \pi = \frac{5\pi}{4}$

7. *[Maximum mark: 7]*

Let $f(x) = px^3 + px^2 + qx$.

(a) Find $f'(x)$. *[2]*

(b) Given that $f'(x) \geq 0$, show that $p^2 \leq 3pq$. *[5]*

a) $3px^2 + 2px + q = f'(x)$ always ≥ 0 so no x intercepts

so $b^2 - 2ac \leq 0$

$(2p)^2 - 4(3pq) \leq 0$

$4p^2 \leq 12pq$

$p^2 \leq 3pq$

*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions in the answer booklet provided. Please start each question on a new page.*

8. *[Maximum mark: 17]*

The line L_1 passes through the points $A(2, 1, 4)$ and $B(1, 1, 5)$.

(a) Show that $\overrightarrow{AB} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$. $\begin{bmatrix} 1-2 \\ 1-1 \\ 5-4 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ *[1]*

(b) Hence, write down

(i) a direction vector for L_1; $\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

(ii) a vector equation for L_1. $r = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix} + t \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ *[3]*

Another line L_2 has equation $\boldsymbol{r} = \begin{pmatrix} 4 \\ 7 \\ -4 \end{pmatrix} + s\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$. The lines L_1 and L_2 intersect at the point P.

(c) Find the coordinates of P. $(4, 1, 2)$ *[6]*

(d) (i) Write down a direction vector for L_2. $\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$

(ii) Hence, find the angle between L_1 and L_2.   *[7]*

$\cos\theta = \dfrac{-1(0) + 0(-1) + 1(1)}{\sqrt{2} \cdot \sqrt{2}} =$

$\cos\theta = \dfrac{1}{2}$

$\theta = 60°$

$4 + 0s = 2 - 1t$
$7 - 1s = 1 + 0t$
$-4 + 1s = 4 + 1t$

$4 = 2 - 1t$
$2 = -t$
$-2 = t$

$7 - s = 1$
$-s = -6$
$s = 6$

$r = \begin{matrix} 2 \\ 1 \\ 4 \end{matrix} + \begin{matrix} -2(-1) \\ -2(0) \\ -2(1) \end{matrix} = \begin{matrix} 4 \\ 1 \\ 2 \end{matrix}$

$\begin{matrix} 4 \\ 7 \\ -4 \end{matrix} + \begin{matrix} 6(0) \\ 6(-1) \\ 6(1) \end{matrix} = \begin{matrix} 4 \\ 1 \\ 2 \end{matrix}$

*Do **NOT** write solutions on this page.*

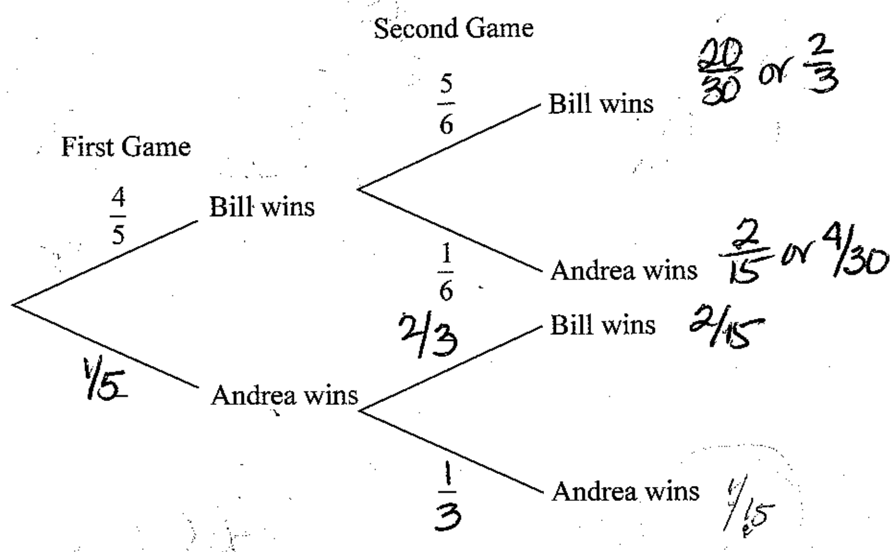
9. *[Maximum mark: 14]*

Bill and Andrea play two games of tennis. The probability that Bill wins the first game is $\frac{4}{5}$.

If Bill wins the first game, the probability that he wins the second game is $\frac{5}{6}$.

If Bill loses the first game, the probability that he wins the second game is $\frac{2}{3}$.

(a) **Copy** and complete the following tree diagram. *(Do **not** write on this page.)* *[3]*



(b) Find the probability that Bill wins the first game and Andrea wins the second game. *[2]*

$4/5 \cdot 1/6 = 4/30$ or $2/15$

(c) Find the probability that Bill wins at least one game. *[4]*

$28/30$

(d) Given that Bill wins at least one game, find the probability that he wins both games. *[5]*

$$P(B|O) = \frac{P(O \cap B)}{P(O)} = \frac{\frac{20}{30}}{\frac{28}{30}} = \frac{20}{28} = \frac{5}{7}$$

*Do **NOT** write solutions on this page.*

10. *[Maximum mark: 15]*

The sides of a square are 16 cm in length. The midpoints of the sides of this square are joined to form a new square and four triangles (diagram 1). The process is repeated twice, as shown in diagrams 2 and 3.

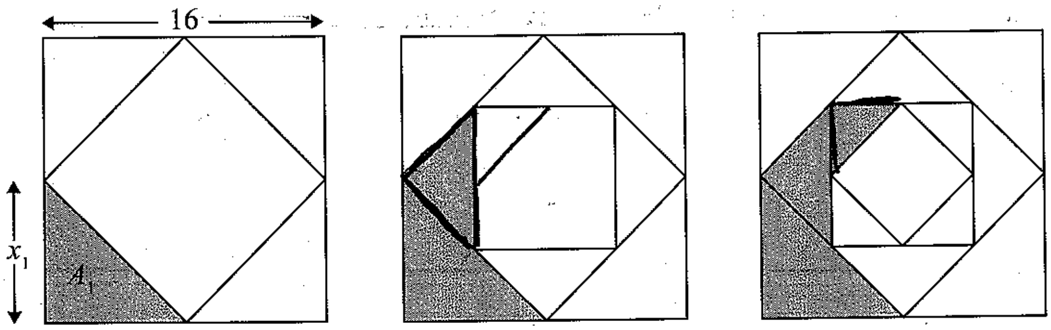


diagram 1   diagram 2   diagram 3

Let x_n denote the length of one of the equal sides of each new triangle.
Let A_n denote the area of each new triangle.

(a) The following table gives the values of x_n and A_n, for $1 \le n \le 3$. **Copy** and complete the table. *(Do **not** write on this page.)* *[4]*

n	1	2	3	4	5	6
x_n	8	$\sqrt{32}$	4	$2\sqrt{2}$	2	$\sqrt{2}$
A_n	32	16	8	4	2	1

(b) The process described above is repeated. Find A_6. *[4]*

$K=2$

(c) Consider an initial square of side length k cm. The process described above is repeated indefinitely. The total area of the shaded regions is k cm². Find the value of k. *[7]*

$U_n = U_1(r)^{n-1}$ $8\left(\frac{\sqrt{2}}{2}\right)^5 = 8\left(\frac{\sqrt{2}}{32}\right)$ $4 \cdot \frac{\sqrt{2}}{2}$ $2\sqrt{2} \cdot \frac{\sqrt{2}}{2}$

$\frac{8}{\sqrt{32}} = \frac{\sqrt{32}}{4}$

$\frac{1}{2}$

$8^2 + 8^2 = c^2$
$128 = c^2$
$= \sqrt{\frac{128}{2}} = \sqrt{32}$

$\frac{8}{4\sqrt{2}} = \frac{2}{\sqrt{2}}$

$\frac{\sqrt{32}}{2}^2 + \frac{\sqrt{32}}{2}^2 = c^2$
$64 = c^2$
$8 + 8 = c^2$
$16 \ne c^2$

$S_\infty = \frac{a}{1-r}$

area $\frac{1}{2}K^2$ so $S_\infty = \frac{\frac{1}{2}\left(\frac{K}{2}\right)^2}{1-\frac{1}{2}} = \frac{\frac{K^2}{84}}{\frac{1}{2}} = \frac{K^2}{}$

$\frac{K^2}{\cancel{8}4} \cdot \frac{\cancel{2}}{1} = \frac{K^2}{4} =$

$X_n = \left(\frac{K}{2}\right)^2 = \frac{K^2}{4}$

Please **do not** write on this page.

Answers written on this page will not be marked.

area of original $= K^2$

$X_1 = \frac{K^2}{4}$

X_2

so $\frac{K^2}{4} = K$

$K^2 = 4K$

$K^2 - 4K = 0$

$K(K-4) = 0$

$K = 4$ or $K = 0$