Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 6]*

Consider the vectors $\boldsymbol{a} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$.

(a) Find

(i) $2\boldsymbol{a} + \boldsymbol{b}$;

(ii) $|2\boldsymbol{a} + \boldsymbol{b}|$. *[4 marks]*

Let $2\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c} = \boldsymbol{0}$, where $\boldsymbol{0}$ is the zero vector.

(b) Find $\boldsymbol{c}$. *[2 marks]*

ai $\begin{pmatrix} 4 \\ -6 \end{pmatrix} + \begin{pmatrix} 1 \\ 4 \end{pmatrix} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$

aii $\sqrt{25+4} = \sqrt{29}$

b) $\begin{pmatrix} 5 \\ -2 \end{pmatrix} + \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$x = -5$ $\quad C = \begin{pmatrix} -5 \\ 2 \end{pmatrix}$

$y = 2$

22137303

International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

MATHEMATICS
STANDARD LEVEL
PAPER 1

Candidate session number

0	0							

Thursday 9 May 2013 (afternoon)

Examination code

2	2	1	3	-	7	3	0	3

1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics SL*** *information booklet* is required for this paper.
- The maximum mark for this examination paper is *[90 marks]*.

$\begin{pmatrix} -15 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} -15 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 11t \end{pmatrix}$ lots of vectors

$(15t)^2 + (22t)^2 = d$

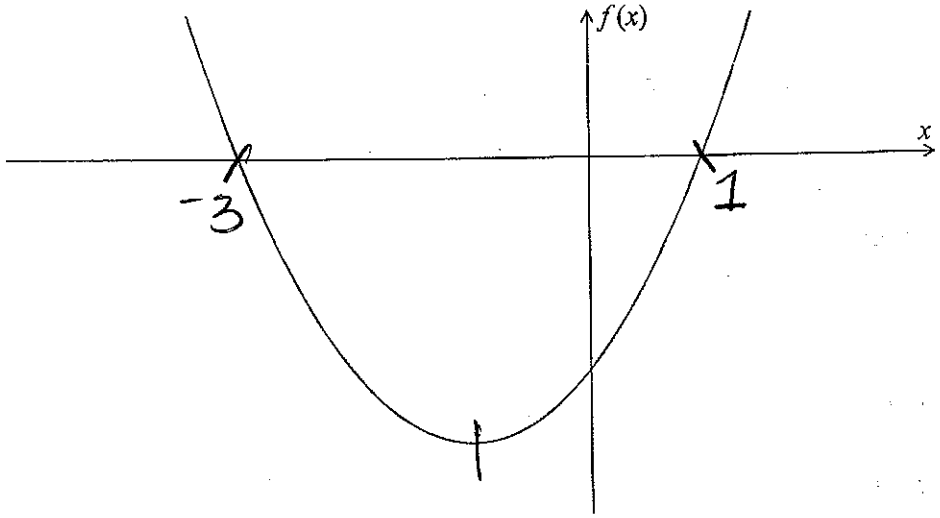
$15t + +15 + (11t)^2$

$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$\sqrt{(-15t)^2+(}$

2. *[Maximum mark: 6]*

The diagram below shows part of the graph of $f(x) = (x-1)(x+3)$.



(a) Write down the x-intercepts of the graph of f. *[2 marks]*

(b) Find the coordinates of the vertex of the graph of f. *[4 marks]*

a) (-3,0) (1,0)

x = -1 y = -4 b) = (-1,-4)

y = (-1-1)(-1+3)

(-2)(2) = -4

3. *[Maximum mark: 7]*

Consider $f(x) = x^2 \sin x$.

(a) Find $f'(x)$. *[4 marks]*

(b) Find the gradient of the curve of f at $x = \frac{\pi}{2}$. *[3 marks]*

$u = x^2$ $\quad v = \sin x$

$u' = 2x$ $\quad v' = \cos x$

$x^2 \cos x + 2x \sin x = f'(x)$

$f'(\pi/2) = (\pi/2)^2 \cos \pi/2 + 2(\pi/2) \sin \pi/2$

$0 + \pi = \pi$

4. *[Maximum mark: 7]*

Let A, B, C and X be square matrices, such that $XA + B = C$.

(a) Find an expression for X in terms of A, B and C. *[2 marks]*

(b) Given that $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 3 \\ -3 & 4 \end{pmatrix}$, find X. *[5 marks]*

5. *[Maximum mark: 6]*

Let $f(x) = \sqrt{x-5}$, for $x \geq 5$.

(a) Find $f^{-1}(2)$. *[3 marks]*

(b) Let g be a function such that g^{-1} exists for all real numbers. Given that $g(30) = 3$, find $(f \circ g^{-1})(3)$. *[3 marks]*

$f^{-1}(x) = x^2 + 5$ $\quad y = \sqrt{x-5}$

$f^{-1}(2) = 2^2 + 5$ $\quad x = \sqrt{y-5}$

$= 9$ $\quad x^2 = y - 5$

$x^2 + 5 = y$

$g(30) = 3$ $\quad g^{-1}(3) = 30$

$\sqrt{30-5} = \sqrt{25} = 5$

6. *[Maximum mark: 6]*

Let $f(x) = \int \frac{12}{2x-5} dx$, for $x > \frac{5}{2}$. The graph of f passes through $(4, 0)$.

Find $f(x)$.

$y = 6\ln(2x-5) + C$

$0 = 6\ln(2(4)-5) + C$

$0 = 6\ln(3) + C$

$C = -6\ln 3$ so $f(x) = 6\ln(2x-5) - 6\ln 3$

7. *[Maximum mark: 7]*

Find the value of

(a) $\log_2 40 - \log_2 5$; *[3 marks]*

(b) $8^{\log_2 5}$. *[4 marks]*

a) $\log_2(40/5) = \log_2 8 = 3$

b) $2^{3\log_2 5}$ $2\log_2 5^3 = 5^3 = 125$

*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions in the answer booklet provided. Please start each question on a new page.*

8. *[Maximum mark: 14]*

Consider points A(1, −2, −1), B(7, −4, 3) and C(1, −2, 3). The line L_1 passes through C and is parallel to $\vec{AB}$.

(a) (i) Find $\vec{AB}$. $\begin{pmatrix} 6 \\ -2 \\ 4 \end{pmatrix}$

(ii) Hence, write down a vector equation for L_1. $r = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + t\begin{pmatrix} 6 \\ -2 \\ 4 \end{pmatrix}$ *[4 marks]*

A second line, L_2, is given by $\boldsymbol{r} = \begin{pmatrix} -1 \\ 2 \\ 15 \end{pmatrix} + s\begin{pmatrix} 3 \\ -3 \\ p \end{pmatrix}$.

(b) Given that L_1 is perpendicular to L_2, show that $p = -6$. dot product of directions *[3 marks]*

(c) The line L_1 intersects the line L_2 at point Q. Find the x-coordinate of Q. *[7 marks]*

$6 \cdot 3 + -2 \cdot -3 + 4 \cdot p = 0$

$18 + 6 + 4p = 0$

$24 + 4p = 0$

$4p = -24$

$p = -6$

$1 + \frac{1}{2}(6) = 4$

$-2 + \frac{1}{2}(-2) = -3$

$3 + \frac{1}{2}(4) = 5$

$1 + 6t = -1 + 3s$

$-2 + -2t = 2 - 3s$

$3 + 4t = 15 - 6s$

$-1 + 4t = 1$

$4t = 2$

$t = 1/2$

*Do **NOT** write solutions on this page.*

9. *[Maximum mark: 16]*

Jar A contains three red marbles and five green marbles. Two marbles are drawn from the jar, one after the other, without replacement.

(a) Find the probability that

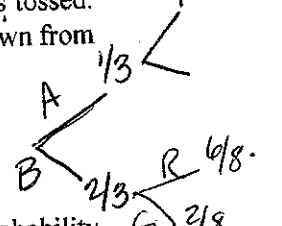
(i) none of the marbles are green; $3/8 \cdot 2/7 = 6/56$

(ii) exactly one marble is green. $(3/8 \cdot 5/7) + (5/8 \cdot 4/7) = \frac{35}{56}$ *[5 marks]*

(b) Find the expected number of green marbles drawn from the jar. *[3 marks]*

$P(0)(0) + P(1)1 + P(2)2 \quad 35/56 + 40/56 = 75/56$

Jar B contains six red marbles and two green marbles. A fair six-sided die is tossed. If the score is 1 or 2, a marble is drawn from jar A. Otherwise, a marble is drawn from jar B.



(c) (i) Write down the probability that the marble is drawn from jar B. $2/3$

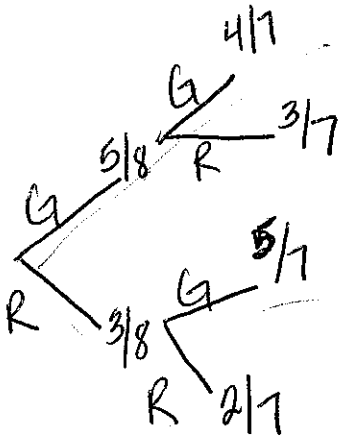
(ii) Given that the marble was drawn from jar B, write down the probability that it is red. *[2 marks]*

$P(R|B) = \frac{P(R \cap B)}{P(B)} = \frac{1/2}{2/3} = 3/4$

(d) Given that the marble is red, find the probability that it was drawn from jar A. *[6 marks]*

$P(A|R) = \frac{P(A \cap R)}{P(R)} = \frac{3/24}{15/24} = 1/5$

$5/8 \cdot 4/7 = \frac{20}{56} + \frac{15}{56} = \frac{35}{56}$



$3/8 \cdot 5/7 = \frac{15}{56}$

*Do **NOT** write solutions on this page.*

10. *[Maximum mark: 15]*

Consider $f(x) = \ln(x^4 + 1)$.

(a) Find the value of $f(0)$. *[2 marks]*

(b) Find the set of values of x for which f is increasing. *[5 marks]*

The second derivative is given by $f''(x) = \frac{4x^2(3-x^4)}{(x^4+1)^2}$.

The equation $f''(x) = 0$ has only three solutions, when $x = 0, \pm\sqrt[4]{3}$ $(\pm 1.316\ldots)$.

(c) (i) Find $f''(1)$.

(ii) **Hence**, show that there is no point of inflexion on the graph of f at $x = 0$. *[5 marks]*

(d) There is a point of inflexion on the graph of f at $x = \sqrt[4]{3}$ $(x = 1.316\ldots)$. Sketch the graph of f, for $x \geq 0$. *[3 marks]*

$A = \begin{pmatrix} 15 \\ 0 \end{pmatrix} + t\begin{pmatrix} -15 \\ 0 \end{pmatrix}$ $B = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + t\begin{pmatrix} 0 \\ 11 \end{pmatrix}$

$\sqrt{(15-15t)^2 + (11t)^2}$

$\sqrt{225 - 30t + 225t^2 + 121t^2}$

450

Please **do not** write on this page.

Answers written on this page will not be marked.