Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 7]*

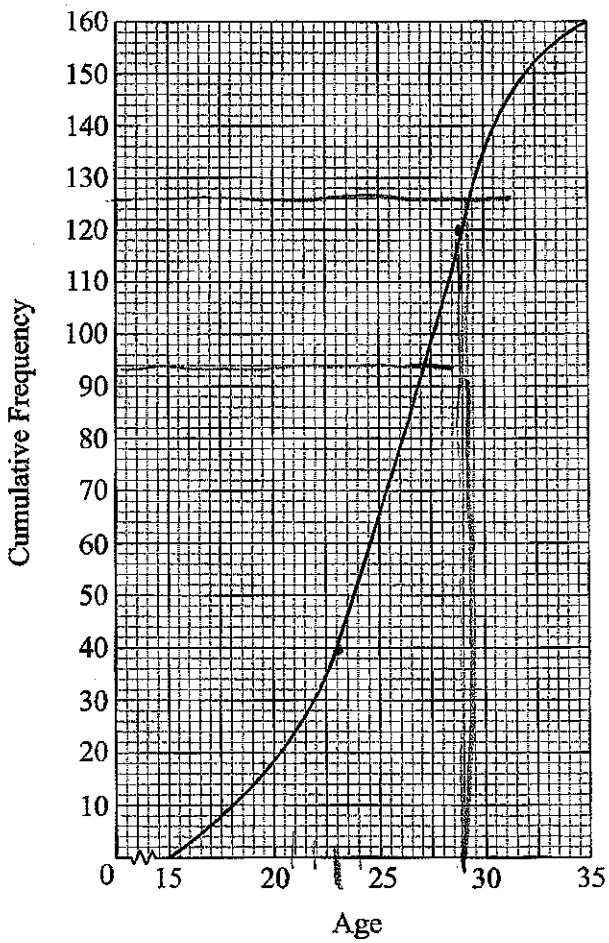
The ages of people attending a music concert are given in the table below.

Age	$15 \le x < 19$	$19 \le x < 23$	$23 \le x < 27$	$27 \le x < 31$	$31 \le x < 35$
Frequency	14	26	52	52	16
Cumulative Frequency	14	40	92	p	160

144

(a) Find p. *[2 marks]*

The cumulative frequency diagram is given below.



(This question continues on the following page)

(Question 1 continued)

(b) Use the diagram to estimate

(i) the 80th percentile;

(ii) the interquartile range.

[5 marks]

p = 144

bi) 29

bii) 23, 28

3. *[Maximum mark: 6]*

Let $f(x) = e^{6x}$.

(a) Write down $f'(x)$. *[1 mark]*

The tangent to the graph of f at the point $P(0, b)$ has gradient m.

(b) (i) Show that $m = 6$.

(ii) Find b. *[4 marks]*

(c) Hence, write down the equation of this tangent. *[1 mark]*

a) $6e^{6x}$ $f'(0) = 6e^{6(0)} = 6$ $m=6$

bii) $f(0) = e^{6(0)} = e^0 = 1$

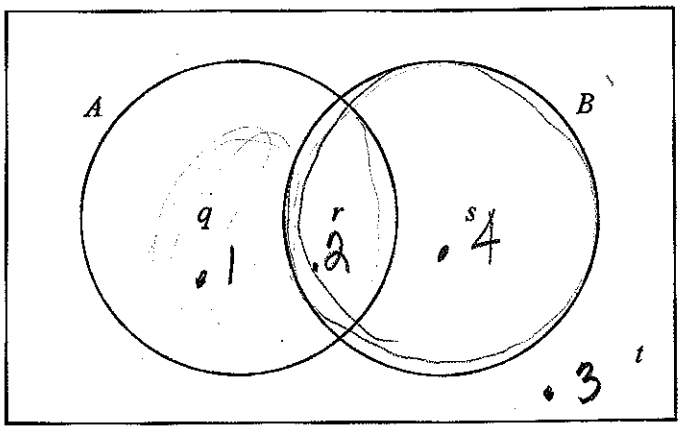
$y - 1 = 6(x - 0)$

$y - 1 = 6x$

$y = 6x + 1$

4. *[Maximum mark: 7]*

Events A and B are such that $\mathrm{P}(A) = 0.3$, $\mathrm{P}(B) = 0.6$ and $\mathrm{P}(A \cup B) = 0.7$.



The values q, r, s and t represent probabilities.

(a) Write down the value of t. *[1 mark]*

(b) (i) Show that $r = 0.2$.

(ii) Write down the value of q and of s. *[3 marks]*

(c) (i) Write down $\mathrm{P}(B')$.

(ii) Find $\mathrm{P}(A|B')$. *[3 marks]*

a) $t = .3$

b) $.3 + .6 - r = .7$

$.9 - r = .7$ $\quad r = .2$

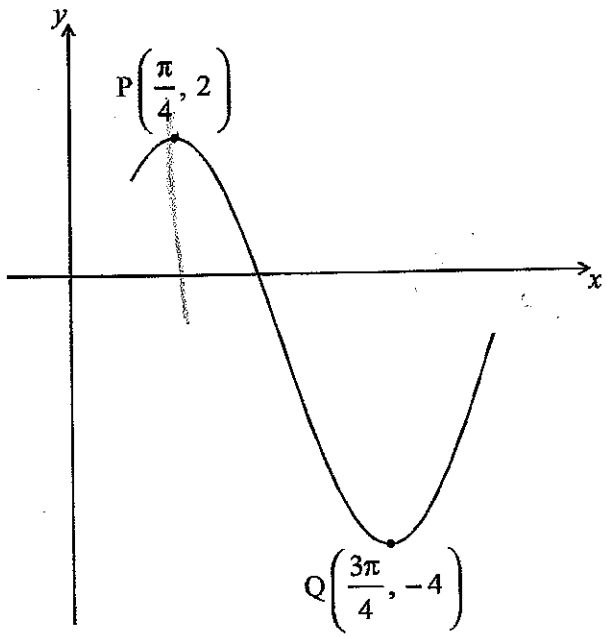
$q = .1$ $\quad s = .4$

c) i) $P(B') = .4$

ii) $P(A|B') = \frac{.1}{.4} = .25$

5. *[Maximum mark: 7]*

The diagram below shows part of the graph of $f(x) = a\cos(b(x-c)) - 1$, where $a > 0$.



The point P$\left(\frac{\pi}{4}, 2\right)$ is a maximum point and the point Q$\left(\frac{3\pi}{4}, -4\right)$ is a minimum point.

(a) Find the value of a. *[2 marks]*

(b) (i) Show that the period of f is π.

(ii) Hence, find the value of b. *[4 marks]*

(c) Given that $0 < c < \pi$, write down the value of c. *[1 mark]*

a) $2 - -4 = 6/2 = 3$

b) i) $2(3\pi/4 - \pi/4) = 2(2\pi/4) = 4\pi/4 = \pi$

ii) $b = 2\pi/\text{period}$ $b = 2\pi/\pi = 2$

c) c = horiz. shift $= \pi/4$

6. *[Maximum mark: 6]*

Given that $\int_0^5 \frac{2}{2x+5}\,dx = \ln k$, find the value of k.

$\ln(2x+5)\Big]_0^5$

$\ln(15) - \ln(5) = \ln\left(\frac{15}{5}\right) = \ln 3$

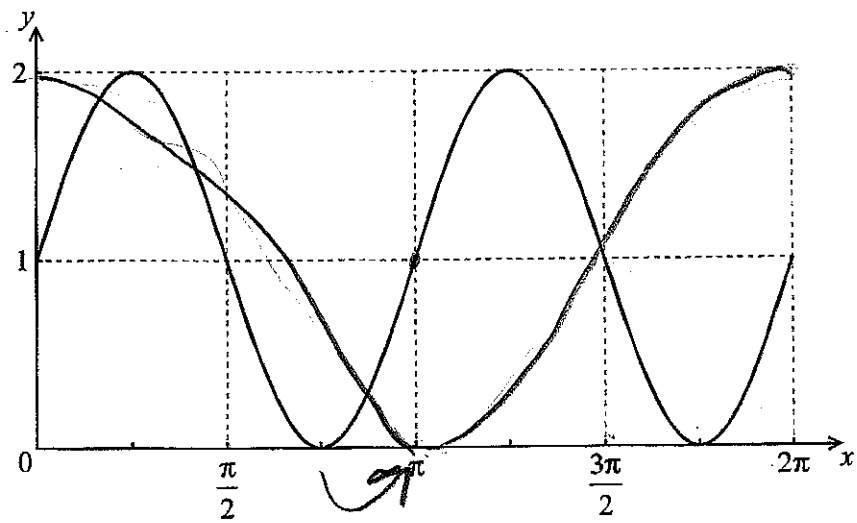
$k = 3$

7. *[Maximum mark: 6]*

Let $f(x) = (\sin x + \cos x)^2$.

(a) Show that $f(x)$ can be expressed as $1 + \sin 2x$. identity *[2 marks]*

The graph of f is shown below for $0 \le x \le 2\pi$.



(b) Let $g(x) = 1 + \cos x$. On the same set of axes, sketch the graph of g for $0 \le x \le 2\pi$. *[2 marks]*

The graph of g can be obtained from the graph of f under a horizontal stretch of scale factor p followed by a translation by the vector $\begin{pmatrix} k \\ 0 \end{pmatrix}$.

(c) Write down the value of p and a possible value of k. $\pi/4$ *[2 marks]*

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*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 17]*

A line L_1 passes though points $P(-1, 6, -1)$ and $Q(0, 4, 1)$.

(a) (i) Show that $\overrightarrow{PQ} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$.

$\begin{bmatrix} 0 - -1 \\ 4 - 6 \\ 1 - -1 \end{bmatrix} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$

(ii) Hence, write down an equation for L_1 in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$. *[3 marks]*

$r = \begin{pmatrix} -1 \\ 6 \\ -1 \end{pmatrix} + t\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$

A second line L_2 has equation $\boldsymbol{r} = \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} + s\begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix}$.

(b) Find the cosine of the angle between PQ and L_2. *[7 marks]*

remember to use direction vectors

(c) The lines L_1 and L_2 intersect at the point R. Find the coordinates of R. *[7 marks]*

$(1, 2, 3)$

$\cos\theta = \dfrac{1 \cdot 3 + -2 \cdot 0 + 2(-4)}{5 \cdot 3} = \dfrac{-5}{15} = -\dfrac{1}{3}$

$-1 + t = 4 + 3s$

$6 - 2t = 2$

$-1 + 2t = -1 - 4s$

$-2t = -4$

$t = 2$

$-1 + 2(1) = 1$

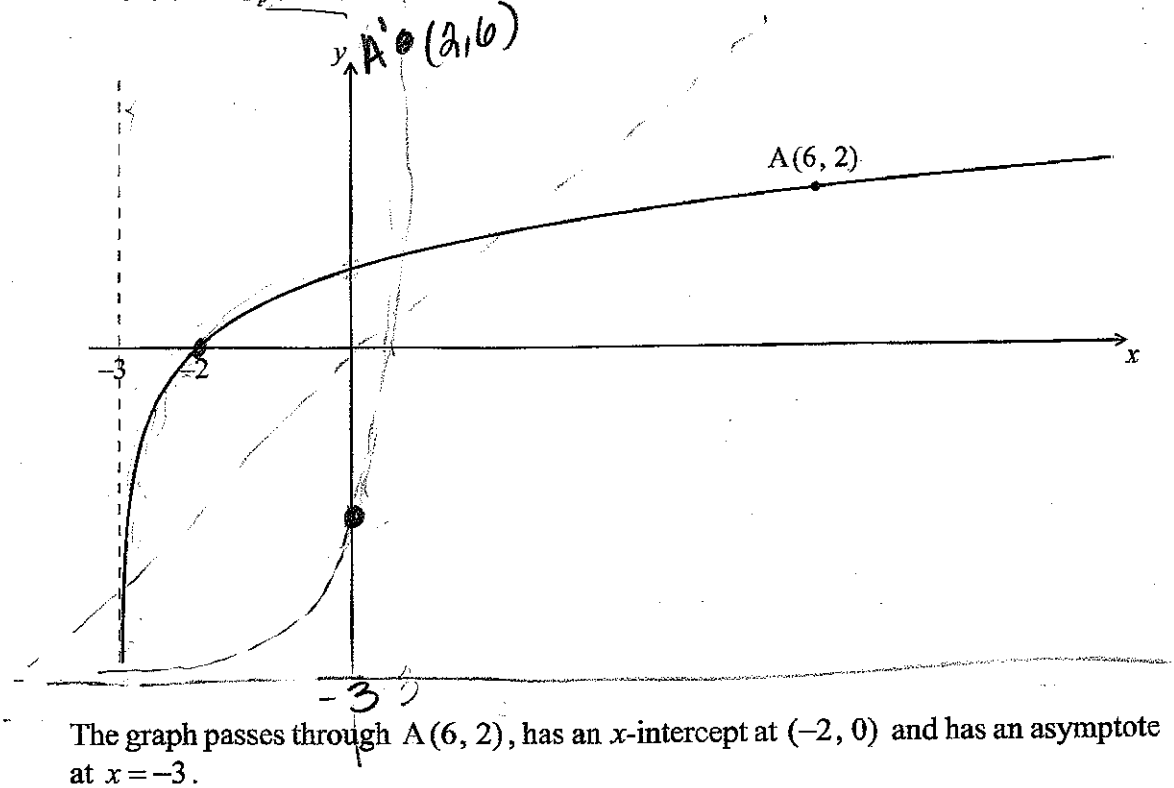
$6 + 2(-2) = 2$

$-1 + 2(2) = 3$

*Do **NOT** write solutions on this page.*

9. *[Maximum mark: 13]*

Let $f(x) = \log_p(x+3)$ for $x > -3$. Part of the graph of f is shown below.



The graph passes through A(6, 2), has an x-intercept at $(-2, 0)$ and has an asymptote at $x = -3$.

(a) Find p. *[4 marks]*

$\log_p(6+3) = 2$

$\log_p(9) = 2 \quad p^2 = 9 \text{ so } p = 3$

The graph of f is reflected in the line $y = x$ to give the graph of g.

(b) (i) Write down the y-intercept of the graph of g. $(0, -2)$

(ii) Sketch the graph of g, noting clearly any asymptotes and the image of A. *[5 marks]*

(c) Find $g(x)$. *[4 marks]*

inverse

$\log_3(x+3) = y \quad \log_3(y+3) = x$

$\cancel{3^{\log_3}}(y+3) = 3^x$

$y+3 = 3^x$

$\boxed{y = 3^x - 3}$

*Do **NOT** write solutions on this page.*

10. *[Maximum mark: 15]*

In this question, you are given that $\cos\frac{\pi}{3}=\frac{1}{2}$, *and* $\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}$.

The displacement of an object from a fixed point, O is given by $s(t)=t-\sin 2t$ for $0\le t\le\pi$.

(a) Find $s'(t)$. $1-2\cos 2t$ *[3 marks]*

In this interval, there are only two values of t for which the object is not moving. One value is $t=\frac{\pi}{6}$.

(b) Find the other value. $1-2\cos 2t=0$ *[4 marks]*

(c) Show that $s'(t)>0$ between these two values of t. *[3 marks]*

(d) Find the distance travelled between these two values of t. *[5 marks]*

$1-2\cos 2t=0$

$1=2\cos 2t$

$\frac{1}{2}=\cos 2t$

$\cos^{-1}\left(\frac{1}{2}\right)=\cancel{\cos^{-1}(\cos}\,2t)$

$\frac{\pi}{3}=2t$

$\frac{\pi}{6}=t$ so also $\frac{5\pi}{6}$

5

$1-2\cos\left(2\frac{\pi}{6}\right)$

$1-2\cos\left(\frac{\pi}{3}\right)$

$1-2\left(\frac{1}{2}\right)$