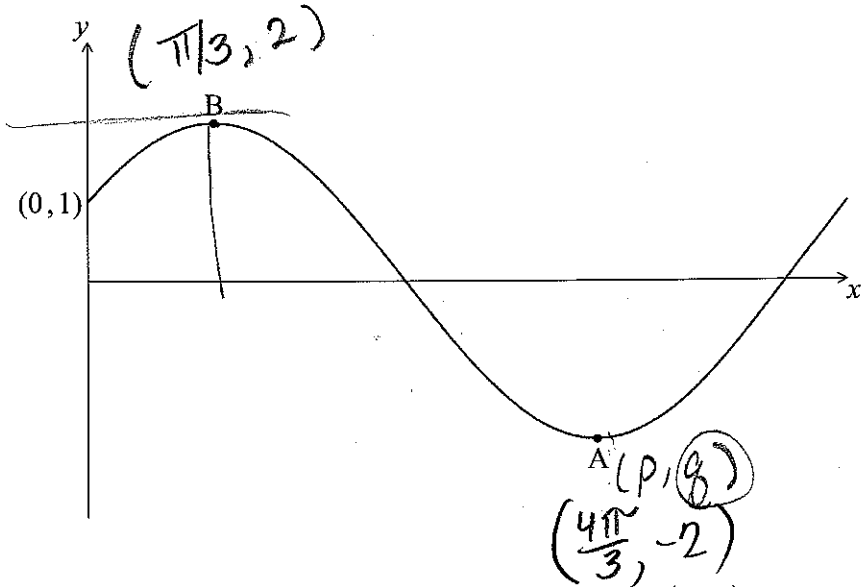
*Do **NOT** write solutions on this page.*

10. *[Maximum mark: 17]*

Let $f(x) = \cos x + \sqrt{3}\sin x$, $0 \le x \le 2\pi$. The following diagram shows the graph of f.



The y-intercept is at $(0,1)$, there is a minimum point at A (p,q) and a maximum point at B.

(a) Find $f'(x)$. *[2 marks]*

(b) Hence

(i) show that $q = -2$; *[10 marks]*

(ii) verify that A is a minimum point.

(c) Find the maximum value of $f(x)$. *[3 marks]*

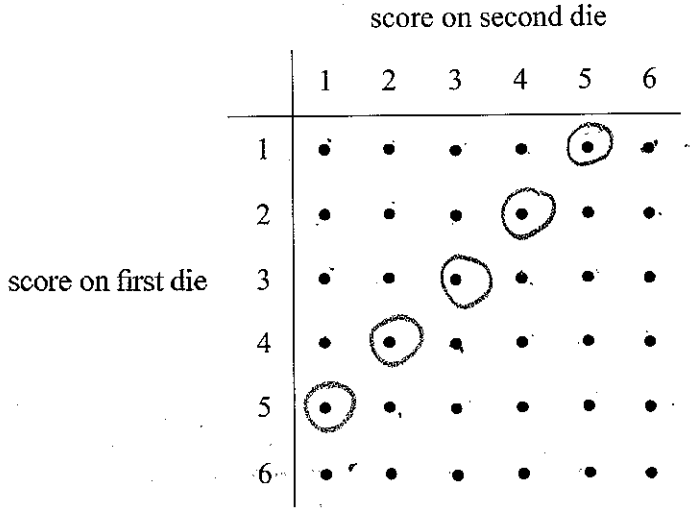
The function $f(x)$ can be written in the form $r\cos(x-a)$.

(d) Write down the value of r and of a. *[2 marks]*

*Do **NOT** write solutions on this page.*

9. *[Maximum mark: 14]*

Two standard six-sided dice are tossed. A diagram representing the sample space is shown below.



Let X be the sum of the scores on the two dice.

(a) (i) Find $P(X = 6)$. 5/36

(ii) Find $P(X > 6)$. 21/36

(iii) Find $P(X = 7 \mid X > 6)$. $\frac{P(=7 \cap >6)}{P(>6)} = \frac{6/36}{21/36} = 6/21$ *[6 marks]*

(b) Elena plays a game where she tosses two dice.

If the sum is 6, she **wins** 3 points.
If the sum is greater than 6, she **wins** 1 point.
If the sum is less than 6, she **loses** k points.

Find the value of k for which the game is fair. *[8 marks]*

$E(V) = 0$

$3(5/36) + 1(21/36) + K(10/36) = 0$

$\frac{15}{36} + 21/36 + 10K/36 = 0$

$15 + 21 + 10K = 0$

$10K = -36$

$K = -3.6$

*Do **NOT** write solutions on this page.*

SECTION B (46 Marks)

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 15]*

Let $f(x) = 3(x+1)^2 - 12$.

$3(x+1)(x+1)-12$ $\quad 3x^2+6x+3-12$

$3(x^2+2x+1)-12$ $\quad 3x^2+6x-9$

(a) Show that $f(x) = 3x^2 + 6x - 9$. *[2 marks]*

(b) For the graph of f

(i) write down the coordinates of the vertex; $(-1, -12)$

(ii) write down the y-intercept; $3(0+1)^2-12 = -9$ $(0, -9)$

(iii) find both x-intercepts. $3(x^2+2x-3)=0$ $\quad 3(x+3)(x-1)$ $(-3, 0)$ $(1, 0)$ *[7 marks]*

(c) **Hence** sketch the graph of f. *[3 marks]*

(d) Let $g(x) = x^2$. The graph of f may be obtained from the graph of g by the following two transformations

a stretch of scale factor t in the y-direction,

followed by a translation of $\begin{pmatrix} p \\ q \end{pmatrix}$.

Write down $\begin{pmatrix} p \\ q \end{pmatrix}$ and the value of t. *[3 marks]*

$t = 3 \quad p = -1 \quad q = -12$

7. *[Maximum mark: 7]*

Given that $f(x) = \frac{1}{x}$, answer the following.

(a) Find the first four derivatives of $f(x)$. *[4 marks]*

(b) Write an expression for $f^{(n)}(x)$ in terms of x and n. *[3 marks]*

$f(x) = x^{-1}$

$f'(x) = -x^{-2}$

$f''(x) = 2x^{-3}$

$f'''(x) = -6x^{-4}$

$f''''(x) = 24x^{-5}$

$f^n(x) = \frac{(-1)^n n!}{x^{n+1}}$

1 × 2

2 × 3

6 × 4

24

6. *[Maximum mark: 7]*

The expression $6\sin x\cos x$ can be expressed in the form $a\sin bx$.

(a) Find the value of a and of b. *[3 marks]*

(b) Hence or otherwise, solve the equation $6\sin x\cos x = \frac{3}{2}$, for $\frac{\pi}{4} \leq x \leq \frac{\pi}{2}$. *[4 marks]*

$\sin 2\theta = 2\sin\theta\cos\theta$ so

$3\sin 2\theta \Rightarrow 3(2\sin\theta\cos\theta) =$

$6\sin\theta\cos\theta$

$\frac{3\sin 2\theta}{3} = \frac{3/2}{3}$

$\sin^{-1}(\sin 2\theta) = \sin^{-1}(1/2)$

$\frac{2\theta}{2} = \frac{\pi/6}{2} + \frac{2k\pi}{2}$ or $\frac{2\theta}{2} = \frac{5\pi}{\frac{6}{2}} + \frac{2k\pi}{2}$

$\theta = \pi/12 + k\pi$ $\quad\quad$ $\theta = 5\pi/12 + k\pi$

so $\theta = 5\pi/12$

5. *[Maximum mark: 7]*

(a) Find $\int \frac{e^x}{1+e^x} dx$. *[3 marks]*

(b) Find $\int \sin 3x \cos 3x \, dx$. *[4 marks]*

a) $\int \frac{e^x}{1+e^x} = \ln(1+e^x) + C$

b) $\int \sin 3x \cos 3x \, dx =$

$\frac{1}{6}(\sin 3x)^2$

4. *[Maximum mark: 5]*

A data set has a mean of 20 and a standard deviation of 6.

(a) Each value in the data set has 10 added to it. Write down the value of

(i) the new mean;

(ii) the new standard deviation. *[2 marks]*

(b) Each value in the original data set is multiplied by 10.

(i) Write down the value of the new mean.

(ii) Find the value of the new variance. *[3 marks]*

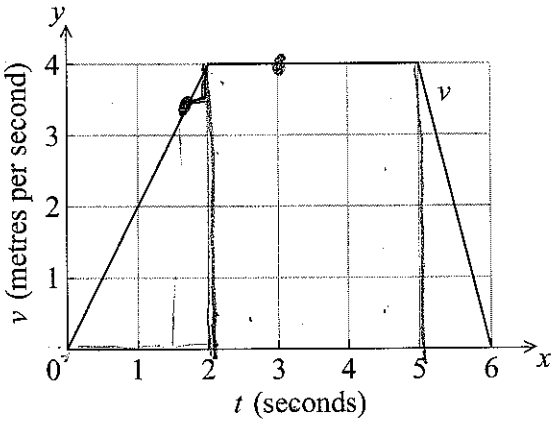
$\mu = 20 \quad \sigma = 6$

a i) $= 30$ ii) $\sigma = 6$ (no change)

bi) ii $\mu = 200$ $\sigma = (60)$ so var $= 60^2 = 3600$

3. *[Maximum mark: 6]*

A toy car travels with velocity $v\ \text{ms}^{-1}$ for six seconds. This is shown in the graph below.



(a) Write down the car's velocity at $t = 3$. *[1 mark]*

(b) Find the car's acceleration at $t = 1.5$. *[2 marks]*

(c) Find the total distance travelled. *[3 marks]*

a) 4ms^{-1}

b) 1 ms^{-1}

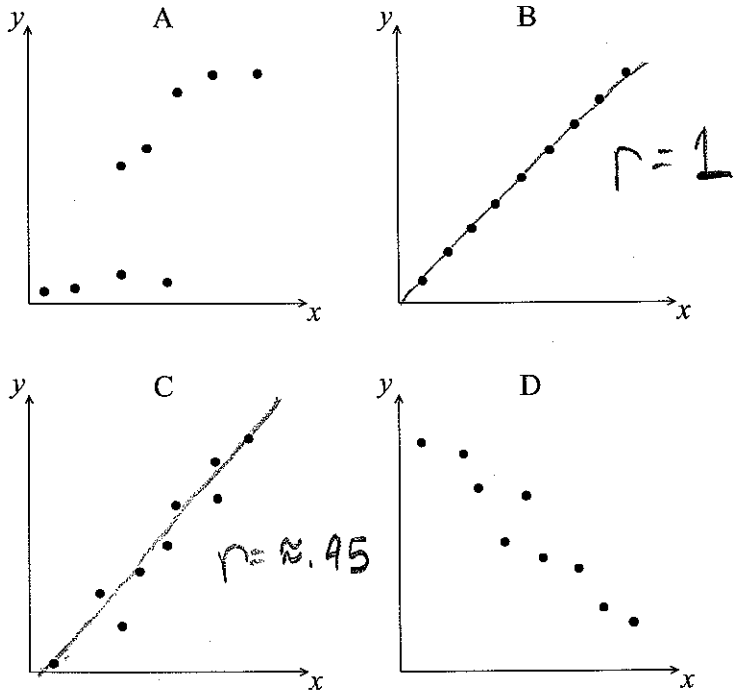
c) 12 + ½(2·4) + ½(1·4)
12 + 4 + 2 = 18

2. *[Maximum mark: 5]*

There are nine books on a shelf. For each book, x is the number of pages, and y is the selling price in pounds (£). Let r be the correlation coefficient.

(a) Write down the possible minimum and maximum values of r. *[2 marks]*

(b) Given that $r = 0.95$, which of the following diagrams best represents the data. *[1 mark]*



(c) For the data in diagram D, which **two** of the following expressions describe the correlation between x and y?

perfect, zero, linear, strong positive, strong negative, weak positive, weak negative *[2 marks]*

a) $-1 \leq r \leq 1$

b) = C

c) Strong negative

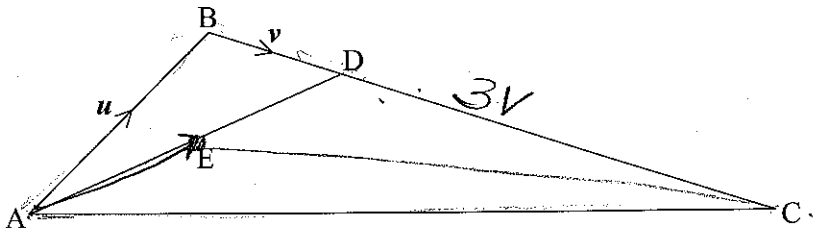
Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A (44 Marks)

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 7]*

In the following diagram, $\boldsymbol{u} = \vec{AB}$ and $\boldsymbol{v} = \vec{BD}$.



The midpoint of $\vec{AD}$ is E and $\frac{BD}{DC} = \frac{1}{3}$.

Express each of the following vectors in terms of $\boldsymbol{u}$ and $\boldsymbol{v}$.

(a) $\vec{AE}$ *[3 marks]*

(b) $\vec{EC}$ *[4 marks]*

$\vec{AE} = \frac{1}{2}(u+v)$

$\vec{EC} = -\frac{1}{2}(u+v) + u + 4v =$

$-\frac{1}{2}u - \frac{1}{2}v + u + 4v = \frac{1}{2}u + 3\frac{1}{2}v$