1. A line L passes through A(1, −1, 2) and is parallel to the line $r = \begin{pmatrix} -2 \\ 1 \\ 5 \end{pmatrix} + s\begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$.

(a) Write down a vector equation for L in the form $r = a + tb$.

$r = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} + t\begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$

$\begin{matrix} 1 \\ -1 \\ 2 \end{matrix} + \begin{matrix} 2 \\ 6 \\ -4 \end{matrix} = \begin{matrix} 3 \\ 5 \\ -2 \end{matrix}$

The line L passes through point P when $t = 2$.

(b) Find

(i) $\overrightarrow{OP}$; $\begin{pmatrix} 3 \\ 5 \\ -2 \end{pmatrix}$

(ii) $|\overrightarrow{OP}|$. $= \sqrt{9+25+4} = \sqrt{38}$

2. Consider the points A (1, 5, 4), B (3, 1, 2) and D (3, k, 2), with (AD) perpendicular to (AB).

(a) Find

(i) $\overrightarrow{AB}$; $\begin{pmatrix} 2 \\ -4 \\ -2 \end{pmatrix}$

(ii) $\overrightarrow{AD}$, giving your answer in terms of k. $\begin{pmatrix} 2 \\ k-5 \\ -2 \end{pmatrix}$

$2 \cdot 2 + -4 \cdot (k-5) + -2 \cdot -2 = 0$

$4 + -4k + 20 + 4 = 0$

$-4k = -28$

$k = 7$

(b) Show that $k = 7$.

The point C is such that $\overrightarrow{BC} = \frac{1}{2}\overrightarrow{AD}$. $\frac{1}{2}AD = \frac{1}{2}\begin{pmatrix} 2 \\ 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$

(c) Find the position vector of C. $\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} + \begin{matrix} 3 \\ 1 \\ 2 \end{matrix} = \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix}$

(d) Find $\cos A\hat{B}C$. $\cos ABC = \frac{BA \cdot BC}{|BA||BC|} =$ $-2 + 4 + -2 = \phi$

3. The vectors $\vec{i}$, $\vec{j}$ are unit vectors along the x-axis and y-axis respectively.
The vectors $\vec{u} = -\vec{i} + 2\vec{j}$ and $\vec{v} = 3\vec{i} + 5\vec{j}$ are given. $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ $\begin{pmatrix} 3 \\ 5 \end{pmatrix}$

(a) Find $\vec{u} + 2\vec{v}$ in terms of $\vec{i}$ and $\vec{j}$. $\begin{pmatrix} -1 \\ 2 \end{pmatrix} + \begin{pmatrix} 6 \\ 10 \end{pmatrix} = \begin{pmatrix} 5 \\ 12 \end{pmatrix}$ $5i + 12j$

A vector $\vec{w}$ has the same direction as $\vec{u} + 2\vec{v}$, and has a magnitude of 26. $\sqrt{5^2 + 12^2} = 13$ so $\times 2$

(b) Find $\vec{w}$ in terms of $\vec{i}$ and $\vec{j}$. $\vec{w} = 10i + 24j$

4. A boat B moves with constant velocity along a straight line. Its velocity vector is given by $v = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$.

$r = \begin{pmatrix} -2 \\ 1 \end{pmatrix} + t\begin{pmatrix} 4 \\ 3 \end{pmatrix}$

At time $t = 0$ it is at the point (−2, 1).

(a) Find the magnitude of v. $\sqrt{4^2 + 3^2} = 5$

(b) Find the coordinates of B when $t = 2$. (6, 7)

(c) Write down a vector equation representing the position of B, giving your answer in the form $r = a + tb$.

5. The following diagram shows quadrilateral ABCD, with $\overrightarrow{AD} = \overrightarrow{BC}$, $\overrightarrow{AB} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ and $\overrightarrow{AC} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$.

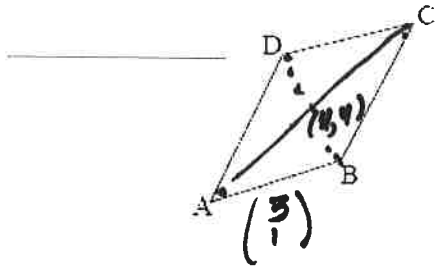

diagram not to scale

(a) Find $\overrightarrow{BC}$. $\overrightarrow{BA} + \overrightarrow{AC} = \begin{pmatrix} -3 \\ -1 \end{pmatrix} + \begin{pmatrix} 4 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

(b) Show that $\overrightarrow{BD} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$. $\overrightarrow{BA} + \overrightarrow{AD} = \begin{pmatrix} -3 \\ -1 \end{pmatrix} + \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$

(c) Show that vectors $\overrightarrow{BD}$ and $\overrightarrow{CA}$ are perpendicular. $-2 \cdot -4 + 2 \cdot -4 = 0$

$8 - 8 = 0$ ✓

6. Line L_1 passes through points A(1, –1, 4) and B(2, –2, 5).

(a) Find $\overrightarrow{AB}$. $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

(b) Find an equation for L_1 in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$. $r = \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix} + t \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

$1 - 3 = -2$
$-1 + 3 = 2$
$4 - 3 = 1$

Line L_2 has equation $\boldsymbol{r} = \begin{pmatrix} 2 \\ 4 \\ 7 \end{pmatrix} + s \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$.

(c) Find the angle between L_1 and L_2. $\cos\theta = \dfrac{2 + -1 + 3}{\dfrac{2 \cdot 1 + -1 \cdot 1 + 1 \cdot 3}{\sqrt{3}\sqrt{14}}} = \dfrac{4}{\sqrt{42}} \approx 51.9^\circ$

(d) The lines L_1 and L_2 intersect at point C. Find the coordinates of C $\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$

$2 + 2s = 1 + t$
$4 + s = -1 - t$ $\quad s = -5 - t$
$7 + 3s = 4 + t$

$2 + 2s = -2$
$2s = -4$
$s = -2$

$7 + 3(-5 - t) = 4 + t$
$7 - 15 - 3t = 4 + t$
$-8 - 3t = 4 + t$
$-12 = 4t$
$-3 = t$

7. Let $\overrightarrow{AB} = \begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix}$ and $\overrightarrow{AC} = \begin{pmatrix} -2 \\ -3 \\ 2 \end{pmatrix}$.

(a) Find $\overrightarrow{BC}$. $\overrightarrow{BA} + \overrightarrow{AC} = \begin{pmatrix} -6 \\ 2 \\ -3 \end{pmatrix} + \begin{pmatrix} -2 \\ -3 \\ 2 \end{pmatrix} = \begin{pmatrix} -8 \\ -1 \\ -1 \end{pmatrix}$.

(b) Find a unit vector in the direction of $\overrightarrow{AB}$. $|AB| = \sqrt{36 + 4 + 9} = \sqrt{49} = 7$ $\quad \frac{1}{7}\begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix}$.

(c) Show that $\overrightarrow{AB}$ is perpendicular to $\overrightarrow{AC}$. ~~$-8 \cdot 6 + -1 \cdot -2 + -1 \cdot -3$~~

$6 \cdot -2 + -2 \cdot -3 + 3 \cdot 2 = 0$ ✓

8. *In this question, distance is in metres.*

Toy airplanes fly in a straight line at a constant speed. Airplane 1 passes through a point A.

Its position, p seconds after it has passed through A, is given by $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \\ 0 \end{pmatrix} + p \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}$

(a) (i) Write down the coordinates of A. $(3, -4, 0)$

(ii) Find the speed of the airplane in m s^{-1}. $\sqrt{4+9+1} = \sqrt{14}$

(b) After seven seconds the airplane passes through a point B.

(i) Find the coordinates of B. $\begin{pmatrix}3\\-4\\0\end{pmatrix} + 7\begin{pmatrix}-2\\3\\1\end{pmatrix} = \begin{pmatrix}-11\\17\\7\end{pmatrix}$ $\rightarrow \overrightarrow{AB} = \begin{pmatrix}-14\\21\\7\end{pmatrix}$

(ii) Find the distance the airplane has travelled during the seven seconds.

$|\overrightarrow{AB}| = \sqrt{(-14)^2+(21)^2+7^2} = \sqrt{686}$ 26.2m

(c) Airplane 2 passes through a point C. Its position q seconds after it passes through C is given by

$$\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}2\\-5\\8\end{pmatrix} + q\begin{pmatrix}-1\\2\\a\end{pmatrix}, a \in \mathbb{R}.$$

$\cos 40^\circ = \frac{-2\cdot-1+3\cdot2+a}{\sqrt{14}\cdot\sqrt{5+a}}$ $.77 = \frac{9+a}{\sqrt{70+14a}}$ $.59(70+14a) = 81+18a+a^2$

$41.5+8.3a = 81+18a+a^2$ $0 = 39.5+9.7a+a^2$ use quad eqn. $\sqrt{70+14a^2}$

The angle between the flight paths of Airplane 1 and Airplane 2 is 40°. Find the two values of a.

$-9.7 \pm \sqrt{9.7^2-4(1)(39.5)}$

9. Let $v = \begin{pmatrix}2\\-3\\6\end{pmatrix}$ and $w = \begin{pmatrix}k\\-2\\4\end{pmatrix}$, for $k > 0$. The angle between v and w is $\frac{\pi}{3}$.

$\frac{1}{2} = \frac{2k+6+24}{7\sqrt{20+k^2}}$

Find the value of k. 18.8 or -4.23

12.25
$3.5(20+k^2) = 4k^2+120k+900$

$245+12.25k^2 = 4k^2+120k+900$

$8.25k^2-120k-655=0$

$\frac{120 \pm \sqrt{120^2-4(8.25)(-655)}}{2(8.25)}$

$\frac{120 \pm 189.8}{16.5}$

10. The vertices of the triangle PQR are defined by the position vectors

$$\overrightarrow{OP} = \begin{pmatrix}4\\-3\\1\end{pmatrix}, \overrightarrow{OQ} = \begin{pmatrix}3\\-1\\2\end{pmatrix} \text{ and } \overrightarrow{OR} = \begin{pmatrix}6\\-1\\5\end{pmatrix}.$$

(a) Find

(i) $\overrightarrow{PQ}$; $\overrightarrow{PO} + \overrightarrow{OQ} = \overrightarrow{PQ} = \begin{pmatrix}-1\\2\\1\end{pmatrix}$

(ii) $\overrightarrow{PR}$. $\begin{matrix}-4\\3\\-1\end{matrix} + \begin{matrix}3\\-1\\2\end{matrix} =$ $\overrightarrow{PO} + \overrightarrow{OR} = \begin{matrix}-4\\3\\-1\end{matrix} + \begin{matrix}6\\-1\\5\end{matrix} = \begin{pmatrix}2\\2\\4\end{pmatrix}$

(b) Show that $\cos R\hat{P}Q = \frac{1}{2}$. $\frac{2\cdot-1+2\cdot2+4\cdot1}{\sqrt{6}\cdot\sqrt{24}} = 6/\sqrt{144} = 6/12 = \frac{1}{2}$

(c) (i) Find $\sin R\hat{P}Q$. $\frac{\sqrt{3}}{2}$

(ii) Hence, find the area of triangle PQR, giving your answer in the form $a\sqrt{3}$. $\frac{1}{2}\sqrt{24}\cdot\sqrt{6}\cdot\frac{\sqrt{3}}{2} =$

$3\sqrt{3}$

11. (a) Let $u = \begin{pmatrix}2\\3\\-1\end{pmatrix}$ and $w = \begin{pmatrix}3\\-1\\p\end{pmatrix}$. Given that u is perpendicular to w, find the value of p.

$2\cdot3+3\cdot-1+-1\cdot p=0$

$6-3-p=0$

$3-p=0$ $3=p$

(b) Let $v = \begin{pmatrix}1\\q\\5\end{pmatrix}$. Given that $|v| = \sqrt{42}$, find the possible values of q.

$42 = 26+q^2$

$16 = q^2$ $q = \pm 4$